5211 – D64 – IVSS – M – 14

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, MAY 2014

MATHEMATICS

Paper : I Sequence and Series

Time : 3 Hours]

[Max. Marks :60

Answer all *questions.*

I. Answer any **five** of the following:

(2 marks each)

1. Define unbounded sequence and give an example.
2. Discuss the convergence of the sequence $\{a_n\}$ where, $a_n = 1+\frac{1}{3}+\frac{1}{3^2}+......+\frac{1}{3^n}$.
3. Define Cauchy's sequence.
4. If ΣU_n and ΣV_n are positive term series and ΣV_n is convergent and there exists a positive constant K such that $U_n < K \cdot V_n$, $\forall_n$. Then prove that ΣU_n is also convergent.
5. Test the convergence of the series $a + b + a^2 + b^2 + a^3 + b^3 + ...$ upto ∞.
6. Test the convergence of the series $\sum \sin\left(\frac{1}{n}\right)$.
7. Using Cauchy's integral test prove $\sum \frac{1}{n^2+1}$ is convergent.
8. Define a conditionally convergent series and give an example.

(5 marks each)

II. Answer any six of the following:

9. If $\{a_n\}$ and $\{b_n\}$ are two convergent sequences. Then, P.T the sequence $\{a_n - b_n\}$ is also convergent.
10. Prove every bounded sequence has at least one limit point.
11. Discuss the convergence of the sequence $\left\{x^{1/n}\right\}$ for $x > 0$.
12. Prove that every sequence has a monotonic sub-sequence.
13. If ΣU_n and ΣV_n are series of positive terms and $\lim_{n\to\infty} \frac{U_n}{V_n} = l$, $(l \neq 0)$ is finite. Then, P.T the series ΣU_n and ΣV_n both converge *Or* diverge together.
14. Test the convergence of $\sum\left[\sqrt[3]{n+1} - \sqrt[3]{n}\right]$.
15. State and prove Cauchy's Root Test for a series of positive terms.

[P.T.O.

16. Test the convergence of the series

$$\frac{x}{1}+\frac{1}{2}\cdot\frac{x^3}{3}+\frac{1}{2}\cdot\frac{3}{4}\frac{x^5}{5}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{x^7}{7}+\ldots\ldots$$

17. Test the convergence of the series.

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+\ldots\ldots$$

18. Test the convergence of the series $\sum\frac{1\cdot5\cdot9\cdots\cdots\cdots(4n-3)}{3\cdot7\cdot11\cdots\cdots\cdots(4n-1)}$

III. Answer any two full quetions of the following: (10 marks each)

19. (a) If the sequence $\{x_n\}$ convergence to l. Then, P.T the sequence $\left\{\frac{1}{n}(x_1+x_2+x_3+\ldots\ldots+x_n)\right\}$ also converges to l.

(b) Prove, $\lim_{n\to\infty}\left(\frac{n^n}{n!}\right)^{\frac{1}{n}}=e$.

20. (a) Prove that $\sum_{n=1}^{\infty}\left(\frac{1}{n^p}\right)$, is convergent if $p>1$, and divergent if $p\le 1$.

(b) Show that the sequence $\{s_n\}$ defined by $S_{n+1}=\sqrt{3S_n}$ and $S_1=1$ is convergent and converges to 3.

21. (a) State and prove Raabe's Test for series of positive terms.

(b) Test the convergence of $\sum\sqrt{\left(\frac{n+1}{n^3+1}\right)}\cdot x^n$.

22. (a) State and prove Leibnitz's test for an alternating series.

(b) Show that, the series $\sum(-1)^n\left(\frac{n+5}{n(n+1)}\right)$ is conditionally convergent.