

THIRD SEMESTER B.Sc. (CS) DEGREE EXAMINATION, NOVEMBER 2018
DISCRETE MATHEMATICAL STRUCTURES

[Max. Marks : 80]

Time : 3 Hours]

Attempt any five questions.
All questions carry equal marks.

1. a) State rule of sum and rule of product with examples.
 (b) Four different mathematics books, five different computer science books and two different control theory books are to be arranged in a shelf. How many arrangements are possible if
 - i) the books in each particular subject must all be together?
 - ii) only mathematics books must be together?
- (c) i) How many committees of 5 with a given chairperson can be selected from 12 persons?
 ii) A total amount of Rs. 1,500 is to be distributed to 3 poor students A, B, C of a class. In how many ways the distribution can be made in multiples of 100 if everyone of these must get at least Rs. 300. (4 + 6 + 6 = 16)
2. (a) Determine the coefficient of $x^2 yz$ in the expansion of $(x - 2y - z)^4$.
 b) Prove that for any three propositions p, q, r , $[p \rightarrow (q \wedge r)] \Leftrightarrow [(p \rightarrow q) \wedge (p \rightarrow r)]$.
 (c) Find the possible truth values of if $[p \wedge (q \wedge r)] \rightarrow (s \vee t)$ is contradiction. (3 + 5 + 4 + 4 = 16)
 (d) State and prove absorption law of logic.
3. a) State any five rules of inference.
 b) Test the validity of

$$p \rightarrow r$$

$$\neg q \rightarrow p$$

$$\neg r$$

(10 + 6 = 16)

$$\therefore q$$

4. a) Define open statement and free variable.
 (b) Write down the negation of at least one parallelogram is not a square.
 c) Explain the method of indirect proof. Using method of direct proof, prove that if m is even integer then $m + 6$ is an even integer. (4 + 4 + 5 + 2 = 16)
 d) Define symmetric difference of 2 sets A and B.

5. a) For any two sets A and B prove that

i) $\overline{A - B} = \overline{A} \cup B$ ii) $A - (A \cap B) = A - B$.

b) Define countable and uncountable sets with example.

c) Out of 30 students in a dormitory, 15 students take art course, 8 take biology course and 6 take chemistry course, further 3 students take all the 3 courses. Find how many students take none of these courses. How many of these take only art course provided 2 students take art and biology course; 3 students take biology and chemistry; 4 student take chemistry and arts course. (4 + 4 + 8 = 16)

6. a) Prove using mathematical induction principle $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$.

b) Define explicit method for describing a sequence. Find an explicit definition of the sequence defined recursively by $a_1 = 7$, $a_n = 2a_{n-1} + 1$ for $n \geq 2$.

c) Find g. c. d. of 81 and 237 and express it in the form $81k + 237l$. (4 + 6 + 6 = 16)

7. a) If $A = \{1, 2, 3, 4\}$

$$R = \{(1, 1) (1, 2) (2, 1) (2, 2) (3, 4) (4, 1)\}$$

$$S = \{(1, 2) (1, 3) (2, 1)\}$$

Write down the matrix relation and draw digraphs of $R \cap S$ and $R \circ S$.

b) If A, B, C are any 3 non empty set prove that

i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

ii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

c) i) If R and S are symmetric prove $R \cup S$ is symmetric.

ii) If R and S are transitive So is $R \cap S$. (6 + 6 + 4 = 16)

8. a) State and prove generalization of pigeonhole principle.

b) Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be any 2 functions then prove if f and g are one-one and onto then g of is also one-one and onto.

c) Let $A = \{1, 2, 3, 4\}$ and f and g are functions from A to A given by

$$f = \{(1, 4) (2, 1) (3, 2) (4, 3)\} \quad g = \{(1, 2) (2, 3) (3, 4) (4, 1)\}$$

each other. (4 + 6 + 6 = 16)