4362 – B02 – IIS BSC(CS) – A – 18

SECOND SEMESTER B.SC. (CS) DEGREE EXAMINATION, APRIL 2018
MATHEMATICS

Time : 3 Hours] [Max. Marks : 80

Instructions : 1) *Answer **any five** questions atleast **one** question from **each** Part.*
2) ***All** questions carry **equal** marks.*

PART – A

1. a) Show that the radius of curvature at any point 'θ' on the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ is $4a\cos\left(\theta/2\right)$.

   b) Verify Rolle's theorem for $f(x) = x^2(1 - x)^2$ in the interval [0, 1].

   c) Obtain the Maclaurin's expansion of $\log(1 + e^x)$ as far as the third degree terms. **(6+5+5=16)**

2. a) Expand e^{ax+by} in neighbourhood of the origin upto the second degree terms.

   b) Evaluate the following limits.

   i) $\lim\limits_{x \to 0} \dfrac{a^x - b^x}{x}$ ii) $\lim\limits_{x \to 1} \left[\dfrac{x}{x-1} - \dfrac{1}{\log x}\right]$

   c) Find the extreme value of $f(x) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

(6+4+6=16)

PART – B

3. a) Evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$.

   b) Evaluate $\int_{-1}^{1} \int_0^z \int_{x-z}^{x+z} (x + y + z)\,dy\,dx\,dz$.

   c) Show that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, $m > 0, n > 0$. **(5+5+6=16)**

[P.T.O.

4. a) Prove that for any vector field A.div. (curl $\vec{A}$) = 0.

   b) If $\vec{A} = xz^3\vec{i} - 2x^2yz\vec{j} + 2yz^4\vec{k}$. Find $\nabla \cdot \vec{A}$, $\nabla \times \vec{A}$ and $\nabla \cdot (\nabla \times \vec{A})$ at (1, –1, 1).

   c) If $F = xy\vec{i} + yz\vec{j} + zx\vec{k}$, evaluate $\int \vec{F} \cdot d\vec{r}$ where c is the curve represented by $x = t$, $y = t^2$, $z = t^3$, $-1 \le t \le 1$. (5+5+6=16)

PART – C

5. a) Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 5e^{-2x}$.

   b) Solve $y'' - 4y' + 13y = \cos 2x$.

   c) Solve $y'' + 3y' + 2y = 12x^2$. (5+5+6=16)

6. a) Solve by method of undetermined coefficients, $y'' + 3y' + 2y = 12x^2$.

   b) Solve by the method of variation of parameters, $y'' + a^2y = \sec ax$.

   c) Solve $x^2y'' - 2y = x^2 + \frac{1}{x}$. (5+6+5=16)

PART – D

7. a) Find the Laplace transform of the following function $f(t) = t^2 \sin at$.

   b) Define periodic function. If f(t) is a periodic function with period T. Show that $L[f(t)] = \frac{1}{1-e^{-ST}} \int_0^T e^{-st} f(t)\,dt$.

   c) Express the following functions interms of unit step function and hence find their Laplace transform $f(t) = \begin{cases} t & 0 < t < 4 \\ s & t > 4 \end{cases}$. (5+6+5=16)

8. a) Find the inverse Laplace transform of the following $\frac{(1-e^{-s})(2-e^{-2s})}{s^3}$.

   b) Verify convolution theorem for the following pair of functions $f(t) = \sin t$ and $g(t) = e^{-t}$.

   c) Solve $y''' + 2y'' - y' - 2y = 0$ given $y(0) = y'(0) = 0$ and $y''(0) = 6$ by using Laplace transform method. (5+5+6=16)