1178 – C64 – IIISS – N – 15

THIRD SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2015

MATHEMATICS

Paper I

(Algebra and Number theory)

(New Syllabus)

Time : 3 Hours] [Max. Marks : 60

Answer **all** *questions.*

. Answer any **five** of the following: $5 \times 2 = 10$

1. Define transitive relation and give an example.
2. Define countability of a set and give an example.
3. Prove that $9^n - 8^n - 1$ is divisible by 8.
4. Find the highest power of 3 contained in 100!.
5. Prove that the identity element in a group is unique.
6. If 'a' is a generator of cyclic group G then a^{-1} is also a generator of cyclic group G.
7. Let $f : G \to G'$ be a homomorphism from group G into G', Prove that if 'e' is an identify in G then f(e)is an identity element in G'.
8. Define Kernal of a homorphism.

. Answer any **six** of the following: $6 \times 5 = 30$

9. If R is an equivalence relation on a set A and $B_a = \{ x \in A : (x, a) \in R \}$ for any $a \in A$, Prove that $\{ B_a : a \in A \}$ forms a partition of A.

10. Prove that every subset of a countable set is countable.

11. Prove that $3^{2n+2} - 8n - 9$ is divisible by 64.

12. If P is a prime number, then show that, $1^{P-1}+2^{P-1}+3^{P-1}+ \ldots\ldots\ldots\ldots\ldots + (P-1)^{P-1} + 1 \equiv 0 (\text{mod } P)$.

13. Prove that the product of 'r' consecutive positive integers is divisible by r!.

14. Prove that a non-empty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H$, $\forall\, a, b, \in H$.

15. Prove that every subgroup of a cycle group is cyclic.

16. Prove that every cycle can be expressed as a product of transpositions in infinitely many ways.

17. State and prove Lagrange's theorem for finite groups.

18. If H is a normal sub group of a group G then the set G/H of all left cosets of H in G form a group with respect to the operation defined by $xH.yH = xyH$, $\forall\, x,y \in G$.

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III. Answer any two of the following questions. 2 × 16

19. a) With usual notations, prove that

i) $\left(\cup\{A_\lambda : \lambda \in \wedge\}\right)^1 = \cap\left\{A_\lambda^1 : \lambda \in \wedge\right\}$

ii) $\left(\cap\{A_\lambda : \lambda \in \wedge\}\right)^1 = \cup\left\{A_\lambda^1 : \lambda \in \wedge\right\}$

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b) Prove that the set Q of rationals is countable.

20. a) State and prove Wilson's theorem.

b) Prove that 28! + 233 is divisible by 899.

21. a) If G and G' are two groups and if $f:G \rightarrow G^1$ is a homomorphism then prove th Kerf = K is a normal subgroup of G.

b) Let $f : G \rightarrow G'$ be a homomorphism of a group G into G′, Prove that

i) If a^{-1} is an inverse of a in G then $f(a^{-1}) = [\, f(a)\,]^{-1}$.

ii) If H is a subgroup of G then f (H) is a subgroup of G′.

22. a) Let N be a normal subgroup of G and Let G/N be the quotient group of N in G. If $f : G \rightarrow G/N$ is a function defined by $f(x) = xN \ \forall\ x \in G$, then Prove that 'f' is homomorphism and Ker f = N.

b) Prove that every infinite cyclic group G is isomorphic to additive group of integers Z