B.A./B.Sc. DEGREE EXAMINATION NOVEMBER 2017.

Third Semester

MATHEMATICS (Optional)

Paper II – ALGEBRA AND ANALYSIS

Time : Three hours | Maximum : 60 marks

Answer **ALL** questions.

I. Answer any **FIVE** of the following : $(5 \times 2 = 10)$

1. Define an abelian ring.

2. Define an integral domain.

3. Prove that in a ring $(R, +, \cdot)$, $a \cdot 0 = 0 \cdot a = 0 \;\forall\, a \in R$.

4. Show that $\lim\limits_{(x,\,y)\to(0,\,0)} \dfrac{xy}{x^2 + y^2}$ does not exists.

5. If $u = \log(\sqrt{x} + \sqrt{y})$, find $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$.

6. If $u = x(1-y)$, $v = x^2 y$, find $\dfrac{\partial(u, v)}{\partial(x, y)}$.

7. Express $\sinh z$ in the form $A \underline{+} iB$.

8. If $x = \cos\theta + i\sin\theta$ then prove that $x^n - \dfrac{1}{x^n} = 2i\sin n\theta$.

II. Answer any **SIX** of the following : $(6 \times 5 = 30)$

9. Show that set of all matrices of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $a, b, c, d \in Z$ the set of all integers is a ring with respect to matrix addition and matrix multiplication but not a commutative ring.

10. Prove that every finite integral domain is a field.

11. Prove that $(R, +, \cdot)$ is a ring without zero divisors iff the cancellation laws holds in R.

12. If a, b, c are any three elements of a ring $(R, +, \cdot)$, show that

 (a) $(-a)\cdot(-b) = ab$

 (b) $a(b-c) = ab - ac$.

13. Find the minimum value of $u = x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = P$.

14. State and prove Euler's theorem for the homogeneous functions of two variables.

15. If $u = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$.

16. With the usual notations, prove that $\frac{\partial(u, v)}{\partial(r, \theta)} = \frac{\partial(u, v)}{\partial(x, y)} \cdot \frac{\partial(x, y)}{\partial(r, \theta)}$.

17. Show that $\log(ix) = \log x + i\frac{\pi}{2}$, where $x \geq 0$.

18. Express $\sin^5\theta$ interms of sines of multiples of θ.

III. Answer any **TWO** of the following : (2 × 10 = 20)

19. (a) Show that every field is an integral domain.

(b) Prove that the set R of numbers of the form $a + b\sqrt{2}$, where a and b are integers is a ring with respect to usual addition and multiplication.

20. (a) If $(R, +, \cdot)$ is a ring with unity such that $(ab)^2 = a^2b^2 \ \forall\ a, b \in R$ then prove that R is commutative.

(b) Find the extreme value of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

21. (a) Explain the Lagrange's method of undetermined multipliers to find the extreme values of $u = f(x, y, z)$ subject to the conditions $f_1(x, y, z) = 0$ and $f_2(x, y, z) = 0$.

(b) If $u = \frac{1}{r}$ and $r^2 = x^2 + y^2 + z^2$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

22. (a) Expand by Maclaurin's theorem $e^x \log(1 + y)$ in powers of x and y as far as the terms of third degree.

(b) If $\sin(A + iB) = x + iy$ prove that $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$ and $\frac{x^2}{\cosh^2 B} - \frac{y^2}{\sinh^2 B} = 1$.

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