SECOND SEMESTER B.A./B.Sc. DEGREE EXAMINATION, APRIL/MAY 2019

Mathematics (Optional)

Paper I— : DIFFERENTIAL CALCULUS

Three Hours

Maximum : 60 Mark

Answer all questions.

Answer any *five* of the following :

1 Find the angle between radius vector and tangent to the curve $r = a \sin \theta$.

2 Write the formula to find the angle of intersection of two polar curves.

3 Show that the curve $y = x^2$ has no points of inflexion.

4 Define : (i) Curvature ; (ii) Evolute.

5 Find the envelope of the curve $A\alpha^2 + B\alpha + C = 0$ where α is the parameter.

6 Find the asymptotes parallel to the co-ordinate axes of the curve $x^2 y^2 - y^2 = 2$.

7 Define : (i) CUSP ; (ii) Isolated point.

8 When the curve $r = f(\theta)$ is symmetric about the initial line.

$(5 \times 2 = 10 \text{ marks})$

Answer any *six* of the following :

9 For the curve $r = f(\theta)$, show that $\tan \phi = r \dfrac{d\theta}{dr}$.

10 Show that the curves $r = a(1 - \sin \theta)$ and $r = a(1 + \sin \theta)$ cut each other orthogonally.

11 Find the pedal equation of the curve $\dfrac{2a}{r} = 1 - \cos \theta$.

12 Find the range of values of x for which the curve $y = 3x^5 - 40x^3 + 3x - 20$ is concave upwards or downwards.

Turn over

13 Show that the equation of the circle of curvature at $\left(\dfrac{a}{4}, \dfrac{a}{4}\right)$ on the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$

$$\left(x - \dfrac{3a}{4}\right)^2 + \left(y - \dfrac{3a}{4}\right)^2 = \dfrac{a^2}{2}.$$

14 Find the evolute of $x = a\cos^3\theta, y = a\sin^3\theta.$

15 Show that for the curve $r^n = a^n\cos n\,\theta$, the radius of curvature, $\int = \dfrac{a^n}{(n+1)\,r^{n-1}}.$

16 Find all asymptotes of the curve $y^3 - x^2y - 2xy^2 + 2x^3 - 7xy + 3y^2 + 2x^2 + 2x + 2y + 1 = 0.$

17 Trace the curve $x^{2/3} + y^{2/3} = a^{2/3},\ a > 0.$

18 Trace the curve $r^2 = a^2\cos 2\,\theta.$

(6 × 5 = 30 mark

II. Answer any *two* of the following :

19 (a) For the curve $r = f(\theta)$, show that $\dfrac{ds}{d\theta} = \sqrt{r^2 + \left(\dfrac{dr}{d\theta}\right)^2}.$

 (b) Find the angle of intersection of the curves $r = a(1 + \sin\theta)$ and $r = b(1 - \sin\theta).$

20 (a) Derive the formula for radius of curvature in pedal form.

 (b) Find the radius of curvature for the curve $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1.$

21 (a) Derive the formula for finding the co-ordinates of the centre of curvature for the cur $y = f(x)$ at any point (x, y) on it.

 (b) Find the envelope of family of lines $\dfrac{x}{a} + \dfrac{y}{b} = 1$ where a and b are connected by the relatio $ab = c^2.$

22 (a) Trace the curve $y^2(a - x) = x^3.$

 (b) Trace the curve $x = a(\theta - \sin\theta)\,;\ y = a(1 - \cos\theta), 0 \le \theta \le 2\pi.$

(2 × 10 = 20 mark