 5052 - B62/63 - IISAS - May - 2014

SECOND SEMESTER B.A./B.Sc. DEGREE EXAMINATION, 2014

Statistics (Optional)

Paper STTH : 2 : CORRELATION & REGRESSION, PROBABILITY DISTRIBUTIONS, ORDER STATISTICS

(New)

Time : Three Hours — Maximum : 80 Marks

Instructions to Candidates:

i) Use of simple / scientific **calculator** is permitted.

ii) Mathematical and statistical **tables** will be supplied on **request.**

iii) New syllabus with effect from **2013 - 2014.**

Part - A

*Answer any **ten** questions. Each question carries 2 marks.* **(10×2=20)**

I. a) Define positive correlation and give one example for it.

b) Show that if one of the regression coefficients is greater than unity, then the other is always less than unity.

c) Write down the plane of regression of X_1 on X_2 and X_3 variates with respective means $\overline{X}_1, \overline{X}_2$ and $\overline{X}_3$ and standard deviations σ_1, σ_2 and σ_3 in a tri-variate distribution.

d) Define multiple correlation coefficient $R_{1.23}$ and give an expression for it in terms of total correlation coefficients r_{12}, r_{13} and r_{23} in a tri-variate distribution.

e) Give the simple linear regression model duly stating the notations involved in it.

f) Define Bernoulli variate. Give its mean and variance.

g) Mention any two examples that follow a Poisson distribution.

h) Write down the p.m.f. of a negative binomial variate. Give any one distinguishing features of this distribution.

i) Define a standard normal variate. Write down its mean and variance.

j) Define Beta-variate of second kind.

k) Define standard Cauchy distribution.

l) Write down the distribution function of $X_{(n)}$.

Part - B

Answer any six questions. Each question carries 5 marks. **(6×5=30)**

II m) Show that Prof. Karl Pearsons coefficient of correlation always lies between -1 and $+1$.

n) The regression lines of Y on X and of X on Y are respectively $a_1X + b_1Y + c_1 = 0$ and $a_2X + b_2Y + c_2 = 0$. Show that

i) Means are $\overline{X} = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \overline{Y} = \dfrac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$

ii) Regression coefficients are $-\dfrac{a_1}{b_1}$ and $-\dfrac{b_2}{a_2}$ respectively.

o) Assuming that the following expression holds true $\left(1 - R^2_{1.23}\right) = \left(1 - r^2_{12}\right)\left(1 - r^2_{13.2}\right)$

Show that : i) $R_{1.23} \geq r_{12}$

ii) If $R_{1.23} = 0$ then $r_{12} = 0$ and $r_{13} = 0$.

p) Write down the normal equations for estimating the parameters involved in a multiple linear regression model $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i \forall\ i = 1, 2, ...n$.

q) Obtain moment generating function and cumulant generating function of a Poisson distribution and show that all its Cumulants are equal.

r) Obtain the rth moment about origin for the following exponential distribution $f(x) = \theta\, e^{-\theta x}, x \geq 0, \theta > 0$ and deduce the recurrence relation for moments $\mu_r^1 = \dfrac{r}{\theta}\mu_{r-1}^1$.

s) Show that for a normal distribution $N\left(\mu, \sigma^2\right)$, mean deviation about mean is $\sqrt{\dfrac{2}{\pi}}\,\sigma$.

t) Define lower order statistic $X_{(1)}$ and derive its probability density function.

Part - C

Answer the following questions. Each question carries 10 marks.

III u) a) Define i) Professor Karl Pearson's coefficient of correlation

ii) Spearman's rank correlation coefficient.

b) X and Y are two correlated variables with $r = r(x, y)$ 'a' and 'b' are two constants, if $U = a_X$ and $V = Y + b$ Show that :

i) $r(u, v) = r(x, y)$

ii) $b_{uv} = a\, b_{xy}$

OR

v) Define partial correlation coefficient $r_{12.3}$ in a tri-variate distribution. Derive an expression for $r_{12.3}$ in terms of total correlation coefficients r_{12}, r_{13} and r_{23} in a tri-variate distribution.

IV. w) Establish the following recurrence relation for moments about mean of a binomial distribution $B(n, p)$ in the usual notations $\mu_{r+1} = pq\left[nr\mu_{r-1} + \frac{d\mu_r}{dp}\right]$.

OR

x) a) Mention any four chief characteristics of a normal distribution.

b) Obtain moment generating function and cumulant generating function of $N(\mu, \sigma^2)$ distribution, and hence find mean and variance of the distribution.

V. y) Define Beta variate of first kind. Obtain the r[th] moment about origin and hence find mean and variance.

OR

z) X and Y are two independent Gamma variates. Show that $U = \frac{x}{x+y}$ and $v = x + y$ are independently distributed U as Beta I variate and V as Gamma variate respectively. **(3×10=30)**