B.A./B.Sc. DEGREE EXAMINATION NOVEMBER 2018.

First Semester

MATHEMATICS (Optional)

Paper II – LINEAR ALGEBRA

ie : Three hours Maximum : 60 marks

Answer **ALL** questions.

Answer any **FIVE** of the following : (5 × 2 = 10)

Define subspace of a vector space.

In any vector space over a field F then show that (i) $0 \cdot x = 0 \ \forall \ x \in V$.

Show that the map $T : V_1(\mathbf{R}) \to V_3(\mathbf{R})$ defined by $T(x) = (3x, 2x, x)$ is a linear transformation.

. Define skew-symmetric determinant and give an example of fourth order.

5. Prove that $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1+x & 1 & 1 \\ 1 & 1 & 1+x & 1 \\ 1 & 1 & 1 & 1+x \end{vmatrix} = x^3$.

6. Define elementary row and column transformations.

7. Find the Minor and Cofactor of 4 in $\begin{bmatrix} -1 & 1 & 0 & -2 \\ -3 & -5 & 2 & 6 \\ 3 & 4 & 0 & -7 \\ 2 & 1 & 1 & 6 \end{bmatrix}$.

8. When the Linear System of equations are said to be consistent and inconsistent?

II. Answer any **SIX** of the following : (6 × 5 = 30)

9. If V is a vector space over a field F then show that Cancellation Laws hold good.

10. Prove that the union of two subspaces of a vector space V over a field F is a subspace iff one is contained in the other.

11. Prove that :

(a) $S \subseteq L(T) \Rightarrow L(S) \subseteq L(T)$

(b) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$

Where S and T are subsets of vector space $V(F)$, $L(S)$ and $L(T)$ are linear span.

12. Show that the set $B = \{(1,1,0)\ (1,0,1)\ (0,1,1)\}$ is a base of vector space $V_3(\mathbf{R})$.

13. If Δ is determinant of order n and Δ' is its reciprocal. Then prove that $\Delta' = \Delta^{n-1}$.

14. Show that $\begin{vmatrix} a & x & y & a \\ x & 0 & 0 & y \\ y & 0 & 0 & x \\ a & y & x & a \end{vmatrix} = \left(x^2 - y^2\right)^2$.

15. Prove that $\rho(A) = \rho(A')$ where A is a matrix.

16. Find the rank of $\begin{vmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 2 & 3 & 1 & 4 \\ 3 & 0 & 2 & 1 \end{vmatrix}$ by elementary transformations.

17. Find the inverse of matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ $R_2 - 2R_1$ $R_3 - 3R_1$ by using elementary row transformations.

18. Solve the system of equations by using elementary transformation,

$$x + y + z = 6$$
$$2x + 3y - 2z = 2$$
$$x - y + z = 2$$

III. Answer any **TWO** of the following :

(2 × 10 = 20)

19. (a) Show that $S = \{(x, y, z) / x - 3y + 4z = 0,\ x, y, z \in \mathbf{R}\}$ is a subspace of $\mathbf{R}^3(\mathbf{R})$.

(b) If a, b, c are linearly independent vectors in a vector space $V(F)$, then show that $a + b$, $a - b$, $a - 2b + c$ are also linearly independent.

(a) Show that a mapping $T: V \to W$ from a vector space $V(F)$ into $W(F)$ is a linear transformation if and only if $T(c_1 x + c_2 y) = c_1 T(x) + c_2 T(y)\ \forall\ c_1 c_2 \in F$ and $x, y \in V$.

(b) Show that $\begin{vmatrix} x+a & b & c & d \\ a & x+b & c & d \\ a & b & x+c & d \\ a & b & c & x+d \end{vmatrix} = x^3(x+a+b+c+d)$.

(a) Prove that Skew-symmetrical determinant of even order is a perfect square.

(b) Find the rank of a matrix by reducing it to normal form,

$$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}.$$

(a) Prove that the interchange of any two rows of a matrix do not alter the rank of a matrix.

(b) For what values of λ and μ, the following system of equations,

$$x + 2y + 3z = 5$$
$$x + 3y - z = 4$$
$$x + 4y + \lambda z = \mu$$ have

(i) no solution

(ii) unique solution

(iii) infinite solution