M.Sc. Fourth Semester Degree Examination

MATHEMATICS — Paper – 4.2

Mathematical Methods

(Old)

Time : 3 Hours] [Max. Marks : 80

Instructions : *1) Answer any **five** questions.*

*2) All questions carry **equal** marks.*

1. (a) Prove that $g_{jk}\, g^{pk} = \delta_j^p$. **(4)**

   (b) Determine the conjugate metric tensor in cylindrical and spherical co-ordinates. **(6)**

   (c) Derive transformation laws for the Christoffel symbols of the first and the second kind. **(6)**

2. (a) State finite Fourier Sine and Cosine transforms. **(4)**

   (b) Find the Fourier transform of e^{-ax^2} where $a > 0$. **(6)**

   (c) Solve $\frac{\partial w}{\partial x} + x\frac{\partial w}{\partial t} = 0$ for $w(x, 0) = 0,\ w(0, t) = t,\ t \geq 0$ using Laplace transform. **(6)**

3. (a) Solve the integral equation $y(x) = 1 + \lambda \int_0^1 x\xi\, y(\xi)\, d\xi$ by the separable kernel. **(8)**

   (b) Solve $g(s) = 1 + \lambda \int_0^1 \sin(s+t)\, g(t)\, dt$ by iterative method. **(8)**

4. (a) Define critical point of an autonomous system. Discuss various types of critical points. **(8)**

   (b) Find all the critical points of the non-linear system

   $$\frac{dx}{dt} = x + 4y - x^2$$

   $$\frac{dy}{dt} = 6x - y + 2xy$$

   and anaylse the nature of these critical points. **(8)**

5. (a) Compute the approximate roots of the equation $x^2 - (3+2\varepsilon)x + (2+\varepsilon) = 0$, for small ε using regular perturbation method. **(8)**

(b) Obtain approximate solutions of the initial value problem

$$y'' = f(x) \cdot y, \; y(0) = 1, \; y'(0) = 1$$

where $f(x)$ is continuous, using perturbation method. **(8)**

6. (a) State different types of models used in O.R. Explain briefly the general methods for solving these O.R. models. **(6)**

(b) Write the canonical and standard form of L.P.P. **(4)**

(c) Use graphical method to solve the following L.P.P.

Minimize $z = 3x_1 + 2x_2$

Subject to

$$5x_1 + x_2 \geq 10$$
$$x_1 + x_2 \geq 6$$
$$x_1 + 4x_2 \geq 12$$

and $x_1, x_2 \geq 0$. **(6)**

7. (a) Use simplex method to solve the following L.P.P.

Maximize $z = 3x_1 + 5x_2 + 4x_3$

Subject to

$$2x_1 + 3x_2 \leq 8$$
$$2x_2 + 5x_3 \leq 10$$
$$3x_1 + 2x_2 + 4x_3 \leq 15$$

and $x_1, x_2, x_3 \geq 0$. **(8)**

(b) Use Big-B method to solve the following L.P.P.

Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$

Subject to

$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 = 10$$

and $x_1, x_2, x_3, x_4 \geq 0$. **(8)**

8. (a) Use the dual simplex method to solve the following L.P.P.

Maximize $z = -2x_1 - x_3$

Subject to

$$x_1 + x_2 - x_3 \geq 5$$
$$x_1 - 2x_2 + 4x_3 \geq 8$$

and $x_1, x_2, x_3 \geq 0$. **(8)**

(b) Prove that the dual of the dual is the primal. **(8)**
