

Roll No. _____

PGIS-N 1022 B-15

M.Sc. Ist Semester (CBCS) Degree Examination
Mathematics

(Ordinary Differential Equations)

Paper - HCT - 1.3

(New)

Time : 3 Hours

Maximum Marks : 80

Instructions to Candidates.

- 1) Solve any five questions
- 2) All questuiions carry equal marks.

1. a) State and prove existence theorem for the solution of the second order intial value problem (8)

b) Let ϕ be any solution of $L(y) = y'' + a_1y' + a_2y = 0$ on an interval I containing a point x_0 then prove that for all x in I $\|\phi(x)\|e^{-k|x-x_0|} \leq \|\phi(x_0)\| \leq \|\phi(x_0)\|e^{k|x-x_0|}$ where $\|\phi(x)\| = [\|\phi(x)\|^2 + \|\phi'(x)\|^2]^{1/2}$ $k = 1 + |a_1| + |a_2|$. (8)
2. a) Let ϕ_1, ϕ_2 be any two linearly independent solutions of $L(y)=0$ on an interval I, then show that every solution ϕ of $L(y)=0$ can be written uniquely as $\phi = c_1\phi_1 + c_2\phi_2$ where c_1, c_2 are constants. (8)

b) Let $\phi_1, \phi_2, \dots, \phi_n$ be n solutions of $L(y)=0$ on an interval I then prove that $W(\phi_1, \phi_2, \dots, \phi_n)(x) = \exp \left[- \int_{x_0}^x a_1(t) dt \right] W(\phi_1, \phi_2, \dots, \phi_n)(x_0)$ (8)
3. a) Define adjoint and self adjoint equation. Further transform the following equations into an equivalent self adjoint equation (4)

b) $x^2 \cdot y'' - 2xy' + 2y = 0$ (4)

c) $f(x) \cdot y'' + g(x) \cdot y' = 0$ (4)

d) $(t^4 + t^2) \frac{d^2x}{dt^2} + 2t^3 \frac{dx}{dt} + 3x = 0$ (4)

4. a) Define characteristic value and characteristic functions of the Sturm-Liouville(S.L) problem (4)

- b) Find the characteristic values and characteristic functions of the S.L problems (6)

i) $\frac{d^2y}{dx^2} + \lambda y = 0, y(0) - y'(0) = 0, y(\pi) - y'(\pi) = 0$

ii) $\frac{d}{dx} \left[x \cdot \frac{dy}{dx} \right] + \frac{\lambda}{x} y = 0, \text{ with } y'(1) = 0, y'(e^{2\pi}) = 0, \lambda \geq 0$ (6)

5. a) Define orthonormal systems with example (3)

- b) Define orthogonal systems with example (3)

- c) Obtain the formal expansion of the function $f(x) = \pi x - x^2, 0 \leq x \leq \pi$ in the series of orthonormal characteristic function $\{\phi_n\}$ of the sturm-Liouville problem

$$\frac{d^2y}{dx^2} + \lambda y = 0, y(0) = 0, y(\pi) = 0 \quad (10)$$

6. a) Let $u(x)$ and $v(x)$ be two solutions of $[r(x)y']' + p(x)y = 0$, such that $u(x)$ and $v(x)$ have a common zero on $a \leq x \leq b$. Then prove that $u(x)$ & $v(x)$ are linearly dependent on $a \leq x \leq b$.

- b) State and prove sturm separation theorem (10)

7. a) Explain Lipschitz condition (4)

- b) Explain Picards method of successive approximations for intial value problems of the type $y' = f(x, y)$ (12)
 $y(x_0) = y_0$
8. a) Derive the method of obtaining general solution of the Riccatti's equation. (6)
- b) Prove that the cross-ratio of any four particular integrals of Riccattis equation is always constant, i.e., independent of x (5)
- c) Explain the method of solving the Riccattis equation $\frac{dy}{dx} = P + Qy + Ry^2$ where P,Q,R are functions of x , when its one particular integral is known (5)
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