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PGIS-N 1029 B-16
M.Sc. Ist Semester Degree Examination
Mathematics
(General Topology)
Paper : HCT-1.5
(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

i) **Answer any five full questions.**

ii) **All questions carry equal marks.**

1. a) Define the usual topology μ on the set of reals R. By using the axioms of a topology show that μ is a topology on R. **(8)**

   b) Define closure of a subset A in a topological space X. Show that $X - \overline{A} = (X - A)^0$. **(8)**

2. a) Define limit point of a set. If A & B are subsets of a space X then prove the followings:

      i) If $A \subset B$ *then* $D(A) \subset D(B)$

      ii) $D(A \cup B) = D(A) \cup D(B)$ **(8)**

   b) Let (x, τ) be a topological space and β is a subfamily of τ. Then prove the following two properties of β are equivalent:

      i) β is a base for τ

      ii) For each $G \in \tau$ and each $p \in G$ there is $U \in \beta$ such that $p \in U \subset G$ **(8)**

3. a) Let f be a mapping of a space x into a space y and s be the subbase for the topology on y then prove the followings are equivalent:

      i) f is continuous.

      ii) The inverse image of each member of s is open in x. **(8)**

   b) If $f : x \to y$ be a continuous function and $A \subset x$ then show that $f / A : A \to y$ is continuous. Is the converse true? Justify. **(8)**

4. a) Prove that a space (x,τ) is a To-space iff for each pair of distinct points $x, y \in x$, $\overline{\{x\}} \neq \overline{\{y\}}$. **(8)**

b) Define a T_2-space. If x is T_2-space and $f: X \to Y$ is a closed bijection then show that Y is a T_2-space. **(8)**

5. a) Define a Regular space. Show that regularity is a topological property. **(8)**

b) Define subsequence of a sequence. Let $f: N \to X$ converges to 'a' in X then every subsequence of f in X converges to 'a'. **(8)**

6. a) Define a connected. Show that any continuous image of a connected space is connected. **(8)**

b) If $\{A_\alpha : \alpha \in D\}$ be a family of connected subsets of a space x such that one of the members of this family intersects every other member then prove that $\bigcup\{A_\alpha ; \alpha \in D\}$ is connected. **(8)**

7. a) Define compact space. Let A be a compact subset of Hausdorff space x and $p \notin A$ then show that there exists disjoint open sets U & V such that $p \in V$ and $A \subset U$ **(8)**

b) Let X be a compact Hausdorff space and Y be an arbitrary space. If $f: X \to Y$ is a continuous closed surjection then prove that Y is also Hausdorff. **(8)**

8. a) Define a metric space. Show that in any metric space, the set of all open spheres is a base for topology on X. **(8)**

b) Define a Lindelof space. Show that every closed subspace of Lindelof space is Lindelof. **(8)**

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