PGIVS-O 1512 A-2K14

M.A./M.Sc. IVth Semester (Non-CBCS) Degree Examination
Mathematics
(Graph Theory-II)
Paper - 4.4
(Old)

Time :3 Hours Maximum Marks : 80

Instructions to Candidates:

1) Answer any **five** questions.
2) All questions carry **equal** marks.

1. a) Show that for a plane graph in which every face is 4-cycle, then q=2p-4. also prove that for a planar graph G with $p \geq 3, q \leq 3p-6$. **(8)**

b) Show that G is outer planar if and only if it has no subgraph homeomorphic to K_4 *or* $K_{2,3}$ **(8)**

2. a) Write steps of simple sequential coloring algorithm with an example. **(10)**

b) Show that every uniquely 4-colorable planar graph is maximal planar. **(6)**

3. a) Write the steps for detection planarity of a graph. Draw all non isomorphic maximal outer planar graphs with six vertices. **(8)**

b) Prove that every tree with two or more vertices is bicolorable. **(8)**

4. a) prove that a map is 2-face colorable if and only if it is an eulerian graph. **(8)**

b) For any nontrivial connected graph G, show that $\alpha_0 + \beta_0 = p = \alpha_1 + \beta_1$ **(8)**

5. a) Prove that the complete graph K_{2n} is 1-factorable. Draw the factors of K_7 **(8)**

b) For every positive integer n, show that k_{2n+1} can be factored into n-Hamiltonian edge disjoint cycles. Draw Hamiltonian cycles of K_7. **(8)**

6. a) Define Hamming distance and prove that a code C is t-error correcting if and only if $d(c) \geq 2t+1$. **(8)**

b) Prove that in a (b,v,r,k, λ) design bk = vr and $\lambda(v-1) = r(k-1)$ **(8)**

7. a) Show that the independent set, clique and vertex cover are NP-complete. **(8)**

b) Prove that $\left\lceil \frac{n}{1+\Delta(G)} \right\rceil \le \Upsilon(G) \le n-\Delta(G)$ **(8)**

8. a) Define total dominating set and number prove that $\left\lceil \frac{diam(G)+1}{3} \right\rceil \le \Upsilon(G)$ **(8)**

b) Show that for any tree T, $\Upsilon(T)=n-\Delta(T)$ if and only if T is a wounded spider with n-vertices. **(8)**