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PGIIIS-N 1529 B-2K13

M.A./M.Sc. IIIrd Semester (CBCS) Degree Examination

Mathematics

(Graph Theory) - I

Paper - HCT-3.2

(New)

Time : 3 Hours   Maximum Marks :80

Instructions to Candidates:

    i)   Answer any **five** fullquestions

    ii)   All questions Carry **equal** marks.

1.   a)   Prove that the number of vertices of odd degree in a graph is always even.   **(5)**

      b)   Determine the number of edges in a graph with 6 vertices, 2 of degree 4 and 4 of degree2. Draw the two such graphs.   **(6)**

      c)   In any connected graph, define the following terms:

         i)   Walk   ii)   Path   iii)   Trail

         iv)   Circuit   v)   Cycle.   **(5)**

2.   a)   Define cut vertex in a connected graph. Show that a vertex 'v' of a connected graph G is a cut vertex of G if and only if there exist vertices u and w $(u, w \neq v)$ such that v is on every u-w path of G.   **(6)**

      b)   Construct cubic graphs with 6 and 8 vertices.   **(4)**

      c)   Show that for any graph G with six vertices G or $\overline{G}$ contains a cycle. Also illustrate through an example.   **(6)**

3.   a)   Define a tree. Show that a (p,q) graph is a tree if and only if it is acyclic and $p = q + 1$ **(6)**

      b)   Define a spanning tree in a connected graph. Show that every connected graph contains a spanning tree.   **(5)**

      c)   Show that a connected graph G is a tree if and only if every edge is a bridge.   **(5)**

4.   a)   Define eccentricity, radius and diameter in a graph. Show that every tree has center consisting of either one vertex or two adjacent vertices.   **(8)**

b) Define Rank and Nullity in a spanning tree. Prove that a graph is a tree if and only if it is minimally connected **(8)**

5. a) Define vertex and edge connectivity of a graph with an example. Prove that in any graph. $K(G) \le \lambda(G) \le \delta(G)$. **(12)**

   b) State the Graphical variations of Menger's theorem. **(4)**

6. a) Let G be a nontrivial connected graph. Then prove that G contains an Eulerian trail if and only if G has exactly two odd vertices. **(8)**

   b) Find under what condition the complete bipartite graph $K_{m,n}$ has an Eulerian graph. **(8)**

7. a) Let G be a graph with $p \ge 3$ vertices and $\delta \ge p/2$ then show that G is Hamiltonian. **(8)**

   b) Prove that a graph H is the block graph of some graph if and only if every block of H is complete. **(8)**

8. a) Show that a graph is the line graph of a tree if and only if it is a connected block graph in which each cut vertex is on exactly two blocks. **(8)**

   b) Prove that every tournament has a spanning path. Explain with an example. **(8)**

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