Roll No. _____

PGIIIS-N 1530 B-16
M.A./M.Sc. IIIrd Semester (CBCS) Degree Examination
Mathematics
(Graph Theory - I)
Paper : HCT 3.2
(New)

Time : 3 Hours

Maximum Marks : 80

Instructions to Candidates:

1) *Answer any five full questions.*

2) *All questions carry equal marks.*

1. a) Prove that the number of vertices of odd degree in a graph is always even. **(5)**

   b) Show that every simple graph must have at least two vertices of the same degree. **(6)**

   c) Prove that a vertex v of a connected graph G is a cutvertex of G if and only if there exists vertices u and w $(u, w \neq v)$ such that v is on every u-w path of G. **(5)**

2. a) Show that a graph G is bipatite if and only if all its cycles are even. **(8)**

   b) Prove that an edge e of a connected graph G is a bridge of G if and only if e does not lie on a cycle of G. Construct r-regular graphs with 8 vertices far each $0 \leq r \leq 4$. **(8)**

3. a) Show that a (p,q) graph G is a tree if and only if it is connected and p = q+1. Also prove that a connected graph G is a tree if and only if every edge is a bridge. **(8)**

   b) Prove that a (p,q) graph G is a tree if and only if it is acyclic and p = q+1. **(8)**

4. a) Define eccentricity, radius and diameter of a graph show that every tree. has either one or two centers. **(8)**

   b) Show that in every network, the value of a maximum flow equals the capacity of a minimum cut. **(8)**

5. a) For any graph G, prove that $K(G) \leq \lambda(G) \leq \delta(G)$; Also construct a graph with $K = 2, \lambda = 3, \delta = 4$ **(8)**

b) Let G be a nontrivial connected graph. Then prove that G contains on Eulerian trail if and only if G has exactly two add vertices. (8)

6. a) Explain the conditions for complete bipourtite graph $k_{m,n}$ has an Eulerian graph. (8)

b) Prove that for any non trivial connected. Eulerian graph G, the set of edges of G can be partitioned into cycles. (8)

7. a) If G is a graph with $p \geq 3$ vertices such that for all non adjacent vertices is and v, $\deg u + \deg v \geq p$, then show that G is Hamiltonian. (8)

b) Prove that in a complete graph with n - vertices there are $\frac{n-1}{2}$ edge disjoint Hamiltonian cycles, if n is an odd number ≥ 3. (8)

8. a) Show that a graph G is the line graph of a tree if and only if it is a connected block graph in which each cut vertex is an exactly two blocks. (8)

b) Prove that every tournament has a Hamiltonian path. (8)

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