

Roll No. \_\_\_\_\_

**PGIIS-N-1016 A-2K13**

**M.A/M.Sc. IInd Semester (CBCS) Degree Examination**

**Mathematics**

**(Complex Analysis)**

**Paper - SCT 2.1**

**(New)**

Time : 3 Hours

Maximum Marks : 80

**Instructions to Candidates:**

- 1) Answer any **five** questions.
- 2) All questions carry **equal** marks.

1. a) Show that the function  $u(x, y) = y^3 - 3x^2y$  is harmonic in  $C$  and determine its harmonic conjugate and the corresponding analytic function. (7)  
b) State and prove Cauchy's theorem for a rectangle. (9)
2. a) Evaluate  $\int_C \frac{dz}{(z^2 + a^2)}$  where  $C$  is a simple closed curve not passing through the point  $z = \pm ai$  (6)  
b) State and prove (10)
  - i) Morera's theorem and
  - ii) Fundamental theorem of algebra. .
3. a) State and prove Laurent's theorem (8)  
b) Represent the function.  $f(z) = (z+1)/(z-1)$  by its Laurents series for the domain.  $|z| < \infty$  (8)
4. a) If a series  $\sum_{n=-\infty}^{\infty} C_n (z-z_0)^n = \sum_{n=0}^{\infty} a_n (z-z_0)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z-z_0)^n}$  Converges to  $f(z)$  at all points in some annular domain about  $z_0$ , then prove that it is the Laurent series expansion for  $f$  in powers of  $(z-z_0)$  for that domain. (8)

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**(1)**

**[Contd....**

- b) Determine the singularities in each of the following functions. Find the singular part if it is a pole. Define  $f(0)$  if it is a removable singularity so that  $f$  is analytic at  $z = 0$ .

i)  $f(z) = (z^2 + 1)/(z(z-1))$

ii)  $f(z) = z \cos\left(\frac{1}{z}\right)$  (8)

5. a) State Cauchy's Residue theorem and evaluate the integral.  $\int_C \frac{(5z-2)}{z(z-1)} dz$

Where  $C$  is the circle  $|z| = 2$  described counter clockwise. (10)

b) Show that  $\int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + 1)^2} dx = \pi/e$  (6)

6. a) State and prove Rouché's theorem (8)

- b) State and prove maximum modulus principle. (8)

7. a) State and prove Riemann mapping theorem. (12)

- b) If  $\{\alpha_k\}$  be sequence of distinct complex numbers such that  $|\alpha_k| \rightarrow \infty$  and  $\{\beta_k\}$  be

another sequence of complex numbers such that  $\sum_{k=1}^{\infty} \frac{|\beta_k|}{|\alpha_k|^{n+1}} < \infty$

Then prove that there exists a meromorphic function  $f$  whose only finite singularities are simple poles at  $\alpha_k$  with residue  $\beta_k$  and the function is represented by.

$$f(z) = \sum_{k=1}^{\infty} \frac{\beta_k}{\alpha_k} \left\langle \frac{z}{\alpha_k}, n \right\rangle$$
 (4)

8. a) State weierstrass factorization theorem and write the application of this theorem to the function  $(\sin \pi z)$  (8)

- b) Write a note on analytic continuation along the path. (8)