Roll No. ______________________ [Total No. of Pages : 2

PGIS-N 1020 B-16

M.A/M.Sc. Ist Semester (CBCS) Degree Examination

Mathematics

(Algebra - I)

Paper : HCT-1.2

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates:

i) **Answer any five full questions.**

ii) **All questions carry equal marks.**

1. a) For a subset A of a group G, prove that A is normal subgroup of G if and only if $N(A) = G$. **(8)**

b) Show that if G is a finite group, then $C_a = \frac{O(G)}{O(N(a))}$. **(8)**

2. a) Suppose G is a finite abelian group and P/O(G), where p is a prime number. Then show that there exists $a \neq e \in G$ such that $a^p = e$ **(8)**

b) If G is the internal direct product of its subgroups $H_1, H_2, ---, H_n$ then show that $G \cong H_1 \times H_2 \times ---- \times H_n$ **(8)**

3. a) Show that any subgroup H of a solvable group G is solvable. **(8)**

b) Prove that a group G is solvable if and only if $\overset{(n)}{G} = (e)$ for some nonnegative integer n. **(8)**

4. a) Show that the quotient filed F of D is the smallest field containing D. **(8)**

b) Prove that any two isomorphic integral domain have isomorphic quotient fields. **(8)**

5. a) Show that ring of integers is a Euclidean ring. **(8)**

b) State and prove unique factorization theorem. **(8)**

6. a) If R is commutative ring with unit element, then prove that R[x] is also commutative ring. If R is an integral domain show that R[x] is also an integral domain. **(8)**

b) State and prove Eisenstein's criterion of irreducibility. **(8)**

7. a) Show that the product of two primitive polynomials over UFD is a primitive poly nomial. **(8)**

b) Prove that an element a of K is algebraic over F if and only if $[F(a):E]$ is finite. **(8)**

8. a) If K is a finite field extension of F and L is a finite field extension of K then show that L is a finite field extension of F and $[L:F]=[L:K][K:F]$. **(8)**

b) Define perfect field Let F be a field of characteristic $p(\neq 0)$. Show that an element a, in some extension of F, is separable over F if and only if F $(a^p) = F(a)$ **(8)**

* * * *