PGIS-N 1020 B-2K12

M.A./M.Sc. First Semester (CBCS) Degree Examination

MATHEMATICS — Paper – HCT 1.2

Algebra – I

(New)

Time : 3 Hours] [Max. Marks : 80

Instructions : *1) Answer any **five** questions.*

*2) All questions carry **equal** marks.*

1. (a) Define conjugacy of a group. Prove that the conjugacy is an equivalence relation in a group. **(8)**

   (b) Let A be a subset of a group G. Show that A is a normal sub-group of G iff $N(A) = G$. **(8)**

2. (a) State and prove Cauchy's theorem for abelian group. **(8)**

   (b) Let A be a subset of a group G. Show that $C(A)$ is a sub-group of G, where $C(A)$ denotes centre of A. **(8)**

3. State and prove three versions of Sylow's theorem. **(16)**

4. (a) Show that every integral domain can be embedded in a field. **(8)**

   (b) The ideal $A = a_0$ is a maximal ideal of the Euclidean ring R iff a_0 is a prime element in R. **(8)**

5. (a) Define unique factorization domain. State and prove unique factorization theorem for Euclidean rings. **(10)**

   (b) Let R be an Euclidean ring and $a, b \in R$ show that $d(a) \le d(a\,b)$; where $b \ne 0$ is not a unit in R. **(6)**

6. (a) State and prove Einstein criterion of irreducibility. **(8)**

   (b) State and prove fundamental theorem on finitely generated modules. **(8)**

7. (a) Prove that a polynomial of degree n over a field has atmost n roots in any extension field. **(8)**

   (b) Let K be an extension field of F then the element $a \in K$ is algebraic over F iff $F(a)$ is a finite extension over F. **(8)**

8. (a) If a field K contains a non-zero integral domain D then R contains a field of quotients of D. **(6)**

   (b) If a is algebraic over F, then show that the field $F(a)$ coincides with the ring $F[a]$. **(5)**

   (c) If an element $a \in K$ is algebraic over F and if $P(x)$ is the minimal polynomial of a over F then show that $F(x) = F(a)$. **(5)**
