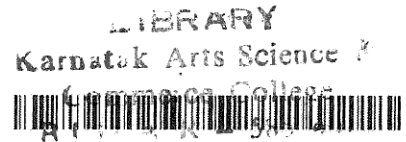


PGIS-N 1020 B-2K12



M.A./M.Sc. First Semester (CBCS) Degree Examination

MATHEMATICS — Paper – HCT 1.2

Algebra – I

(New)

Time : 3 Hours]

[Max. Marks : 80

Instructions : 1) Answer any **five** questions.

2) All questions carry **equal** marks.

1. (a) Define conjugacy of a group. Prove that the conjugacy is an equivalence relation in a group. (8)
(b) Let A be a subset of a group G . Show that A is a normal sub-group of G iff $N(A) = G$. (8)
2. (a) State and prove Cauchy's theorem for abelian group. (8)
(b) Let A be a subset of a group G . Show that $C(A)$ is a sub-group of G , where $C(A)$ denotes centre of A . (8)
3. State and prove three versions of Sylow's theorem. (16)
4. (a) Show that every integral domain can be embedded in a field. (8)
(b) The ideal $A = a_0$ is a maximal ideal of the Euclidean ring R iff a_0 is a prime element in R . (8)
5. (a) Define unique factorization domain. State and prove unique factorization theorem for Euclidean rings. (10)
(b) Let R be an Euclidean ring and $a, b \in R$ show that $d(a) \leq d(ab)$; where $b \neq 0$ is not a unit in R . (6)
6. (a) State and prove Einstein criterion of irreducibility. (8)
(b) State and prove fundamental theorem on finitely generated modules. (8)

PGIS-N 1020 B-2K12



7. (a) Prove that a polynomial of degree n over a field has atmost n roots in any extension field. (8)
- (b) Let K be an extension field of F then the element $a \in K$ is algebraic over F iff $F(a)$ is a finite extension over F . (8)
8. (a) If a field K contains a non-zero integral domain D then K contains a field of quotients of D . (6)
- (b) If a is algebraic over F , then show that the field $F(a)$ coincides with the ring $F[a]$. (5)
- (c) If an element $a \in K$ is algebraic over F and if $P(x)$ is the minimal polynomial of a over F then show that $F(x) = F(a)$. (5)
-