

SIIS-N-195 B-17

B.Sc./B.A. IIIrd Semester Degree Examination

MATHEMATICS

(Groups, Rings, Line and Multiple Integral)

Paper - 3.1

(New)

Time : 3 Hours

Maximum Marks : 60

Instructions to Candidates:

Answer all the questions.

SECTION - A

I. Answer any TEN of the following:

(10 × 2 = 20)

- 1) Show that the multiplicative group of cube roots of unity is cyclic.
- 2) Find the orders of all the elements of an additive group of integer module 6 ie $(\mathbb{Z}_6; +6)$.
- 3) Show that every cyclic group is abelian.
- 4) Find all the right cosets of the subgroup $H = \{0,3\}$ in the group $(\mathbb{Z}_6; +6)$.
- 5) Define Homomorphism and Endomorphism.
- 6) Show that the mapping $f : (\mathbb{R}, +) \rightarrow (\mathbb{R}^+, \cdot)$ defined by $f(x) = e^x \forall x \in \mathbb{R}$ is a homomorphism.
- 7) Write the closure law, Associative law and distributive laws under Multiplication Operation of a Ring.
- 8) Define cancellation laws of Ring.
- 9) Evaluate $\int_C (2y + x^2) dx + (3x - y) dy$ along the, curve $x = 2t, y = t^2 + 3$ where $0 \leq t \leq 1$.
- 10) Evaluate $\int_C (x + y) dx + (y - x) dy$ along the parabola $y^2 = x$ from (1,1) to (4,2).

11) Show that $\int_0^1 \int_0^2 (x+y) dy dx = 3$.

12) Evaluate $\int_0^3 \int_1^2 (x^2 + 3y^2) dy dx$.

SECTION - B

II. Answer any FIVE of the following:

(5 × 5 = 25)

- 1) Prove that Intersection of two subgroups is also a subgroup.
- 2) The order of an element 'a' of a group is the same as that of its inverse a^{-1} . i.e. $o(a) = o(a^{-1})$.
- 3) Let G be a finite group and H be any subgroup of G then $o(H) \mid o(G)$.
- 4) Prove that the set G/H of all cosets of a normal subgroup H of G group G is a group under the Binary operation defined by $H_a H_b = H_{ab} \forall H_a, H_b \in G/H$.
- 5) If f is a homomorphism of G into \bar{G} then
 - i) $f(e) = e'$ where e and e' are Identities of G and G'.
 - ii) $f(x^{-1}) = [f(x)]^{-1} \forall x \in G$.
- 6) If $G = \left\{ \begin{bmatrix} a & o \\ o & o \end{bmatrix} \mid a \in R^* \right\}$, where $R^* = R - (0)$ Then show that $f : (G, \cdot) \rightarrow R^*$ defined by $f \left(\begin{bmatrix} a & o \\ o & o \end{bmatrix} \right) = a \forall \begin{bmatrix} a & o \\ o & o \end{bmatrix} \in G$ is an Isomorphism.
- 7) A ring without zero divisors if and only if cancellation laws holds in it.

8) The Necessary and sufficient condition for a non empty subset S of a ring R to be a subring are.

i). $a \in s, b \in s \Rightarrow a - b \in s$

ii) $a \in s, b \in s \Rightarrow a \cdot b \in s$

SECTION - C

III. Answer any **THREE** of the following:

(3 × 5 = 15)

1) Evaluate $\int_C (x^2 - 2xy) dx + (x^2 y - 3) dy$. Where C is the boundary of the region defined by $y^2 = 8x$ and $x = 2$.

2) Evaluate $\int_C (x + y + z) ds$ where C is the line joining the points (1,2,3) and (4,5,6).

3) Evaluate $\iint_R x^2 y^2 dx dy$ where R is the circle $x^2 + y^2 = 1$.

4) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz dx dy dz$.

5) Show that $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} dx dy dz = (e-1)^3$.

