

SVIS 332 A-2K13

B.Sc. VIth Semester Degree Examination
Mathematics
(Complex Analysis and Improper Integrals)
Paper - 6.2

Time : 3 Hours

Maximum Marks : 80

Instructions to Candidates.

Answer all the sections.

Section - A

Answer any ten of the following.

(10×2= 20)

1. Prove that $\cos 6\theta = 1 - 18\cos^2\theta + 48\cos^4\theta - 32\cos^6\theta$
2. Prove that $\cosh 3x = 4\cosh^3 x - 3\cosh x$.
3. Separate into real and imaginary parts of $\sec(x+iy)$.
4. Find all the values of $\log(\sqrt{3}+i)$.
5. Show that $\left(\frac{z-3}{z+3}\right)^2 = 2$ represents a circle.
6. If a function $f(z)$ is differentiable at $z = z_0$, then show that it is continuous at that point.
7. Show that $f(z) = z - \bar{z}$ is not differentiable.
8. Evaluate $\int_C \frac{e^{2z}}{(z-3)^3} dz$ where C is $|z|=4$
9. Discuss the convergence of $\int_1^\infty \frac{1}{(1+x)\sqrt{x}} dx$.
10. Evaluate $\int_0^\infty \left(-\frac{9}{2}\right)$

11. Prove that $\beta(a,b) \cdot \beta(a+b,c) = \beta(b,c) \cdot \beta(a,b+c)$.

12. Evaluate $\int_0^\infty \frac{x^4(1-x^5)}{(1+x)^{15}} dx$.

Section - B

Answer any five of the following:

(5×6= 30)

13. State and prove the necessary condition for a function $f(z)$ to be analytic in a domain D.

14. Construct the analytic function $f(z) = u + iv$, whose imaginary part is

$$x^3y - xy^3 + x \cdot y + x + y.$$

15. Find the real part of the analytic function, whose imaginary part is $r^2 \cdot \sin 2\theta + r \cdot \cos \theta$ and hence the corresponding analytic function.

16. If $f(z) = u + iv$ is analytic function. Then show that

$$\left\{ \frac{\partial}{\partial x} |f(z)| \right\}^2 + \left\{ \frac{\partial}{\partial y} |f(z)| \right\}^2 = |f'(z)|^2$$

17. State and prove Cauchy's Integral formulae for the function $f(z)$ in the domain D.

C

18. Evaluate $\int_C z^2 dz$, Where C is

a) The straight line joining the points (0,0) & (3,1)

b) The path consisting of the two line segment joining (0,0) to (3,0) and (3,0) to (3,1).

19. Evaluate $\int_C \frac{\sin \pi z + \cos \pi z}{(z-1)^2 \cdot (z-3)} dz$ where C is the circle $|z|=2$.

Section - C

Answer any **five** of the following

($5 \times 6 = 30$)

20. Expand $\cos^5 \theta \cdot \sin^3 \theta$ in terms of sine multiple of θ .

21. Sum the series

$$x \cdot S \sin \alpha + \frac{x^2}{2!} S \sin 2\alpha + \frac{x^3}{3!} S \sin 3\alpha + \dots \text{to } \infty$$

22. Define convergence of an improper integral. Discuss the convergence of Gamma function.

23. Define Beta and Gamma function. Find the relation between them.

24. Prove that, $\beta(m, n) = 2 \cdot \int_0^{\pi/2} \sin^{2m-1} \theta \cdot \cos^{2n-1} \theta \cdot d\theta$ and hence evaluate $\int_0^{\pi/2} \cos^{10} \theta \cdot d\theta$.

25. Evaluate $\int_2^6 \frac{1}{\sqrt{(x-2)(5-x)}} dx$.

26. Prove that

$$\int_0^1 \frac{x^2}{\sqrt{1-x^4}} dx \int_0^1 \frac{1}{\sqrt{1+x^4}} dx = \frac{\pi}{4\sqrt{2}}$$

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