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SVIS-N-330-A-18
B.Sc. VIth Semester Degree Examination
MATHEMATICS
(Theory of Graphs)
paper-6.3
(New)

Time : 3 Hours Maximum Marks : 80

Instructions to candidates:
Answer all the sections

Section-A

I. Answer any **Ten** of the following **(10×2 = 20)**

1) Define Vertex cut set
2) Show that if G is a connected graph and U is an edge cut set with λ (G) edges, then G-U contains exactly two components.
3) Show that if Cp is cycle with $p \geq 3$ then K(Cp)=2
4) Define Eulerian graph and give an example.
5) For which positive integers n, K_n is eulerian graph.
6) Define Hamiltonian cycle and Hamiltonian graph.
7) Give an example of a graph which eulerian and also Hamiltonian.
8) Show that $K_{2,4}$ is planar graph.
9) Show that if e is an edge of K_5 then $K_5 - e$ is planar graph.
10) How many edges are in a maximal outer planar with 25 vertices.
11) Give two different 2-chromatic graphs.
12) Find the chromatic number of a complete graph Kp with $P \geq 1$ vertices.

Section-B

II. Solve any **Five** of the following **(5× 6= 30)**

1) i) Find the connectivity of $K_{m,n}$ where $1 \leq m \leq n$

   ii) Show that vertex connectivity of non trivial tree in one.

2) Let G be a graph with K(G)=1 what are the possible values for the following numbers

   i) K(G-e) ii) K(G-V)

   iii) λ(G-V) iv) λ(G-e)

3) Which of the following graphs are separable or non-separable

   i) K_{14} ii) K_{22}

   iii) P_7 iv) C_6

4) Show that a connected graph G is eulerian iff the set of edges of G can be partitioned in to cycles.

5) For which positive integers m and n $K_{m,n}$ is eulerian.

6) Prove that a graph is Hamiltonian if and only if its closure is Hamiltonian.

7) i) Give an example of graph G with 5 vertices such that G and $\bar{G}$ are Hamiltonian.

   ii) Show that every Hamiltonian graph has no cut vertex.

8) If a graph G is Hamiltonian then prove that for every non empty proper subset S of V(G) $K(G-S) \leq |S|$, where K(G-S) is the number of components in G-S.

Section-C

III. Solve any **Five** of the following : **(5× 6= 30)**

1) Let G be a connected planar graph with p vertices, q edges and r regions then show that q-p+2=r.

2) If G is (p,q) planar graph in which every region is an n-cycles then prove that

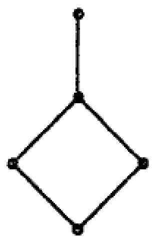
   i) $q = \frac{n}{2} r$

   ii) $q = \frac{n(p-2)}{n-2}$

3) Show that a graph is planar if and only if each of its blocks is a planar.

4) If G is a connected planar graph with p-vertices q-edges and r-regions and if its dual has p* vertices, q* edges and r* regions then show that p*=r, q*=q , r*=p

5) Define a coloring of a graph give three different coloring of the following graph. Find X(G)



6) Show that a tree with at least two vertices is bi-chromatic (or 2-chromatic)

7) Show that a graph with at least one edge is bi chromatic if and only if it has no cycles of odd length.

8) Give two different graphs such that each satisfies $X(G) = \Delta(G) + 1$