

Roll No. \_\_\_\_\_

[Total No. of Pages : 4

**SVIS 327 A-2K14**  
**B.Sc. VIth Semester Degree Examination**  
**Mathematics**  
**(Graph Theory - II)**  
**Paper - 6.3 (d)**

Time :3 Hours

Maximum Marks : 80

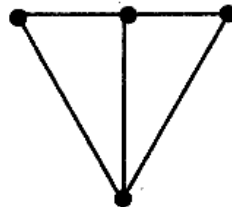
*Instructions to Candidates:-*  
*Answer all sections.*

**Section - A**

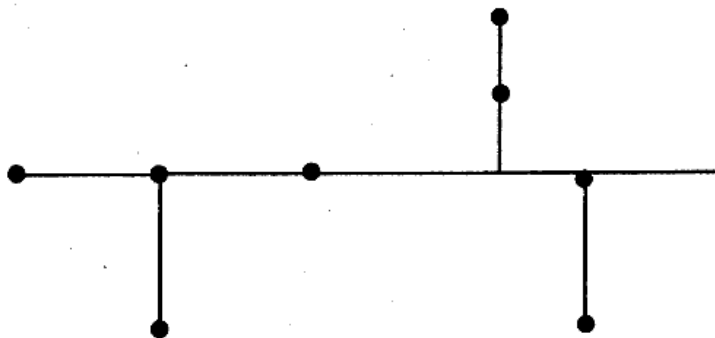
Answer any **10** of the following :

**(10×2=20)**

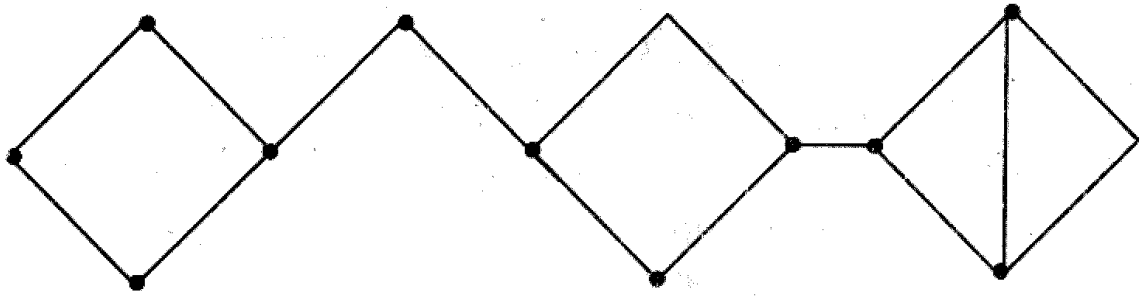
- I. Define a tree and draw a tree with Six vertices.
- II. Define a spanning tree and draw three spanning trees of the following graph



- III. Find the number of Cut-Vertices of the following graph



IV. Find the number of Blocks of the following graph.



V. Prove that  $K_{n,n+1}$  is "Non Hamiltonian"  $\forall n \geq 1$ .

VI. Let  $F_1 = (V_1, E_1)$  be a forest of 7 trees, where  $|E_1| = 40$  Find  $|V_1|$

VII. State Mengers theorem.

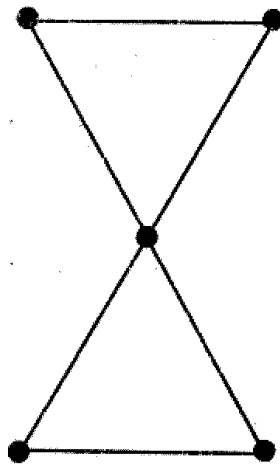
VIII. Show that Hamiltonian path is spanning tree.

IX. State Konigsberg's Bridge problem with figure.

X. Prove that  $K_6$  graph is Hamiltonian cycle but not Eulerian.

XI. How many vertices are in a tree with 26 edges.

XII. Define Eulerian path verify the following graph has an Eulerian cycle.



**Section - B**

Answer any 5 of the following :

(5×6=30)

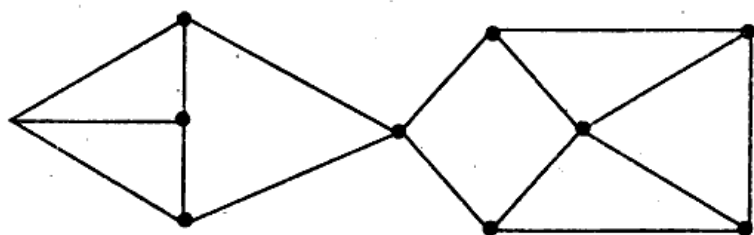
- I. Prove that every edge is a Bridge if and only if every spanning tree of  $G$  contains an edge.
- II. a) Define the terms Bridge, Block and Cut - Vertices.  
b) Give an example of a graph with 6 vertices which has Cut-Vertex but not contain a bridge.
- III. Let  $T$  be a tree with 50 edges. The removal of certain edges from  $T$  yields two different trees  $T_1$  &  $T_2$  Given that the number of vertices in  $T_1$  equals the number of edges in  $T_2$  Find the number of vertices and edges in  $T_1$  &  $T_2$ .
- IV. Prove that a  $(p, q)$  graph is a tree if and only if it is acyclic and  $p = q + 1$ .
- V. i) Give an example of a graph which is both Eulerian and Hamiltonian.  
ii) Give an example of a graph with 4 vertices which is Hamiltonian but not Eulerian.  
iii) Give an example of a graph  $G$  such that  $G$  &  $\bar{G}$  are both Eulerian and Hamiltonian.
- VI. Define a Hamiltonian-Path. Draw a complete graph  $K_7$  Find there are three mutually edge-disjoint Hamiltonian cycles and hence draw them.
- VII. Explain the "Travelling Sales Man" problem.

**Section - C**

Answer any 5 of the following : <http://www.karnatakastudy.com>

(5×6=30)

- I. Write the Whitney's Inequality. Find  $X(G)$ ,  $\lambda(G)$  and  $\delta(G)$  of the graph



Hence verify the Whitney's Inequality.

- II. Let  $G$  be a graph of order  $P \geq 3$  If  $\deg(V) \geq P/2 \forall$  Vertex of  $G$  then prove that  $G$  is Hamiltonian.

- III.** Show that if a  $(p, q)$  graph is a forest with  $K$  components then prove that  $q = p - k$ .
- IV.** In a Binary tree on  $n$  vertices show that the number of Pendent vertices is equal to  $\frac{n+1}{2}$   
Is it possible to draw a tree with 5 vertices having degree 2, 2, 3, 3, 5.
- V.** Prove that a connected graph  $G$  is Eulerian if and only if it can be decomposed into Circuits.
- VI.** Prove that every block of a connected graph  $G$  is Eulerian then  $G$  is Eulerian.
- VII.** Define a binary tree. Show that the number of vertices in a binary tree is odd.