Roll No. ______________________________ [Total No

SVS 338 B-14

B.Sc. Vth Semester Degree Examination

Mathematics

(Vector Analysis and Laplace Transformations)

Paper - 5.1

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates :

Answer all the sections.

SECTION - A **(10×2=20)**

Answer any ten of the following

1. Find the unit normal vector at the point (1 , -1, 2) to the surface $x^2y + y^2z + z^2x = 5$

2. Find $\nabla\phi$ where $\phi(x,y,z) = (x^2 + y^2 + z^2)e^{-(x^2+y^2+z^2)^{1/2}}$

3. Find $\nabla.\vec{f}$ where $\vec{f} = 3x^2 i + 5xy^2 j + xyz^3 k$ at (1, 2, 3)

4. Show that $Curl(grad\,\phi) = 0$

5. Find the Fourier coefficient a_0 for the function $f(x) = 3x^2$; $-\pi < x < \pi$ and $f(x+2\pi) = f(x)$

6. Define periodic function , Give an example.

7. Find the Fourier coefficient a_n for $f(x) = x - x^2$; $-1 < x < 1$

8. Show that $L[af(t) + bg(t)] = aL[f(t)] + bL[g(t)]$

9. Evaluate $L[\cos 3t \sin 2t]$

10. Verify the convolution theorem for $f(t) = 1$ and $g(t) = \sin t$

11. Find the inverse Laplace transform of $\frac{s+1}{s^2 + 2s - 8}$

12. Using the convolution theorem find $L^{-1}\left[\frac{1}{s(s^2+1)}\right]$

SECTION - B

Answer any **FIVE** of the following. **(5×6=30)**

1. Find the angle between the normals to the surface $xy - z^2 = 0$ at the points (1,9,-3) and (-2,-2,2).

2. Show that $div(\vec{f}\times\vec{g}) = \vec{g}.(curl\,\vec{f}) - \vec{f}.(curl\,\vec{g})$

3. Show that $\nabla\times(\nabla\times\vec{f}) = \nabla(\nabla.\vec{f}) - \nabla^2\vec{f}$

4. Verify Green's theorem for $\oint_c [(xy+y^2)\,dx + x^2\,dy]$ Where C is the closed curve bounded by $y = x$ and $y = x^2$

5. Obtain the Fourier series expansion of $f(x) = x + \frac{x^2}{4}; -\pi < x < \pi$

6. Find the Fourier series for the function $f(x) = \begin{cases} -1; -3 < x < 0 \\ 0; \quad x = 0 \\ 1\ ; 0 < x < 3 \end{cases}$

7. Obtain the half - range sine series for $f(x) = x; 0 < x < \pi$.

SECTION - C

Answer any **FIVE** of the following. **(5×6=30)**

1. Obtain the Laplace transform of t^n, where n is a positive integer. Hence evaluate $L[t^2 - 3t + 5]$

2. If f(t) is a periodic function with the period T show that $L[f(t)] = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\,dt$.

Further find $L[e^{-t}]$ $for\ 0 \le t < 2$ & f(t+2) = f(t)

3. Find the inverse Laplace Transform of $\frac{1}{s(s^2+3s+2)}$

4. Show that $L^{-1}\left[\frac{s^2}{s^4+4a^2}\right]=\frac{1}{2a}[\cosh at \sin at+\sinh at \cos at]$

5. State and prove convolution theorem.

6. Find $L\left[\frac{\sin at-\sin bt}{t}\right]$

7. Solve $y''-9y=-8e^t$ given that $y(0)=0$ and $y'(0)=10$.