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SVS 338 B - 15
B.Sc. Vth Semester Degree Examination
Mathematics
(Vector Analysis and Laplace Transformation)
Paper -5.1

Time :3 Hours Maximum Marks : 80

Instructions to Candidates :

Answer **all** the Sections

Section - A

Answer Any **TEN** of the following **(10×2 =20)**

1. Find the directional derivative of $f(x,y,z) = xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of vector $i+2j+2k$

2. If, $uF = \nabla V$ where u, V are scalar fields and F is a vector field. Show that. $F.Curl\ F = 0$

3. If $\vec{f}\ \&\ \vec{g}$ are irrotational. Show that $\vec{f} \times \vec{g}$ is solenoidal

4. Show that, the area bounded by a simple closed curve C is given by $\oint_C (xdy - ydx)$

5. Find the Fourier coefficient a_0 for the function, $f(x) = x - x^2$ from, $x = -\pi\ to\ \pi$

6. Find the Fourier coefficient a_n if $f(x) = |x|$, where $-\pi < x < \pi$

7. Find the half range fourier coefficient b_n, for the function $f(x) = \begin{cases} x & , \quad 0 < x \le \frac{\pi}{2} \\ \pi - x & , \quad \frac{\pi}{2} < x \le \pi \end{cases}$

8. Evaluate, $L[Cosh\ at - \cos at]$

9. Find the Laplace transform of $e^{-2t}\cosh 4t$

10. Verify convolution theorem for the functions, $f(t)=t$ and $g(t)=e^{t}$

11. Find the inverse Laplace transform of $\frac{S+b}{S^2+a^2}$

12. Let, f(t) be of exponential order having derivative, which is continuous and . If

$L[F(t)]=f(S)$, then prove that $L\left[F^{1}(t)\right]=Sf(S)-F(0)$

Section - B

Answer Any **Five** of the following **(5×6 = 30)**

1. Find the constants a&b such that the surfaces $3x^2-2y^2-3z^2+8=0$ is orthogonal to $ax^2+y^2=bz$ at the point $(-1,2,1)$

2. Show that $\nabla.\left[r\nabla\left(\frac{1}{r^3}\right)\right]=\frac{3}{r^4}$, Where $r=x^2+y^2+z^2$

3. Prove that, $div\left(\vec{f}\times\vec{g}\right)=\vec{g}\,curl\vec{f}-\vec{f}curl\vec{g}$

4. Evaluate by using stokes theorem $\oint_C(\sin zdx-\cos xdy+\sin ydz)$ where C is the boundary of the rectangle $0\le x\le\pi, 0\le y\le 1, z=3$

5. Obtain the Fourier Series for $f(x)=e^{-x}$ in the interval $0<x<2\pi$

6. Find the Fourier series for the following function over the interval $(-3,+3)$

$$f(x)=\begin{cases}1+2x\ , & -3<x<0\\ 1-2x\ , & 0<x<3\end{cases}$$

7. Expand $f(x)=\frac{1}{4}-x, if\ 0<x<\frac{1}{2}$ in the Fourier series of sine terms

$$=x-\frac{3}{4}, if\ \frac{1}{2}<x<1$$

Section - C

Answer Any **Five** of the following **(5×6 = 30)**

1. Find the Laplace transformation of

   a) $\sin^2 t$

   b) $e^{2t}\cos^2 t$

2. Evaluate, $L\left[\frac{\cos 2t - \cos 3t}{t}\right]$

3. Find f(t), if $L[f(t)] = \frac{s+3}{(s^2+6s+13)^2}$

4. Evaluate $L^{-1}\left[\frac{1}{s^2(s^2+1)(s^2+9)}\right]$

5. Find the inverse Laplace transform of $\frac{1}{s^2(s^2-a^2)}$ by using the convolution theorem

6. Solve, $\frac{d^2y}{dt^2} + 9y = 18t$ where, $y(0) = 0, y(\pi/2) = 0$ by using Laplace transformation

7. Express the function in terms of unit step function and their Laplace transformation
$$f(t) = \begin{cases} 2t , & 0 < t < \pi \\ 1 , & t > \pi \end{cases}$$