

Roll No. \_\_\_\_\_

[Total No. of Pages : 4

SVS-N- 340- B-18

B.Sc Vth Semester Degree Examination

MATHEMATICS

(Graph Theory - I)

Paper -5.3

(New)

Time : 3 Hours

Maximum Marks : 80

*Instruction to Candidates:*

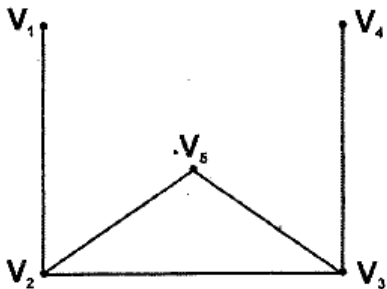
*Answer all the sections.*

**SECTION -A**

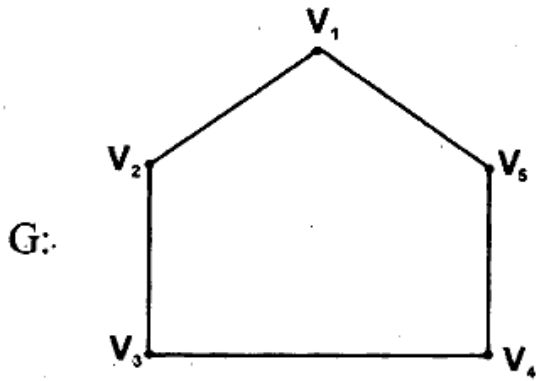
I. Answer any TEN of the following.

(10×2=20)

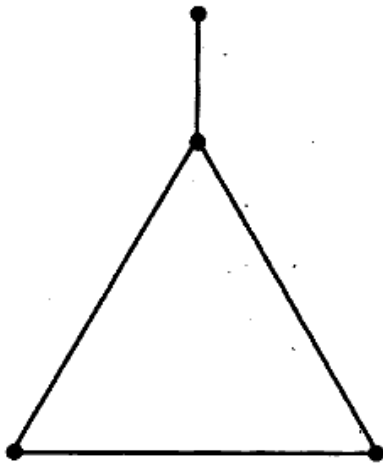
1. Define the term multigraph and pseudograph.
2. Define order and size of a graph G.
3. Define a null graph. Draw a null graph with 4 vertices.
4. Define odd vertex and even vertex.
5. Find the degree of each vertex minimum and maximum degree of the graph shown below.



6. Write the complete of graph.



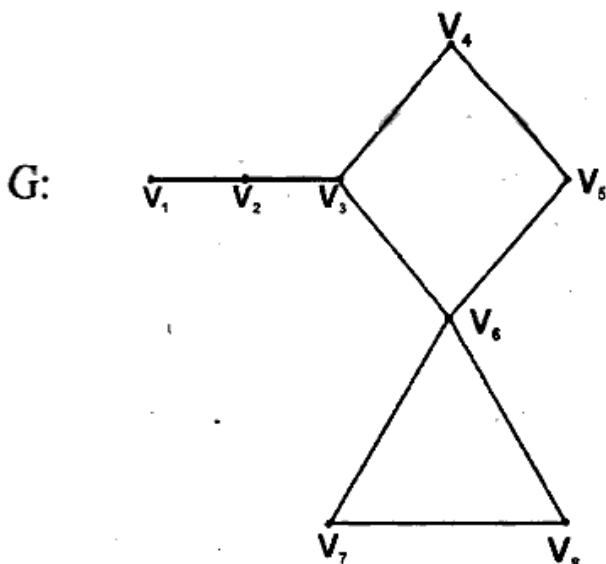
7. Draw the line graph  $L(G)$  of the graph G.



8. Show that  $\overline{K}_p$  is a spanning sub graph  $k_p$ .

9. How many vertices and edges are there in complete bipartite graph  $k_{7,11}$ .

10. Find all the cut vertices in G.



11. Show that every edge of the  $p_3$  is a bridge.
12. Draw all the trees with 6 vertices and  $\Delta(T) \geq 6$ .

### SECTION - B

II. Answer any FIVE of the following.

(5×6=30)

1. Show that the number of edges of an  $r$ -regular graph with  $p$ -vertices is  $q = \frac{pr}{2}$ . Does there exist a 4-regular graph with 5 vertices? If so, construct it.
2. Prove that every self-complementary graph has  $4n$  or  $4n+1$  vertices.
3. If  $G$  is a  $(p, q)$  graph whose vertices have degree  $d_i$ , then show that  $L(G)$  has  $q$ -vertices and  $q_L$  edges where  $q_L = \sum_{i=1}^p d_i^2 - q$ .
4. Show that the removal of a vertex  $v$  from  $K_p$  ( $p \geq 2$ ) produces a complete graph  $K_{p-1}$ .
5. Show that a closed walk of odd length contains a cycle.
6. Prove that a graph  $G$  with  $p$ -vertices and  $\delta(G) \geq \frac{p-1}{2}$  is connected.
7. Show that a non-trivial graph is bipartite if and only if all of its cycles are even.
8. If a graph has exactly two vertices of odd degree then there exists a path joining these two vertices. <http://www.karnatakastudy.com>

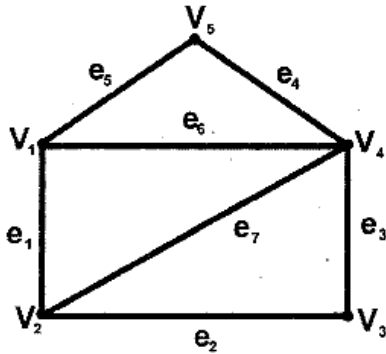
### SECTION - C

III. Answer any FIVE of the following.

(5×6=30)

1. If  $G$  is a tree. Then show that every distinct vertices of  $G$  are joined by a unique path in  $G$ .
2. Define eccentricity  $e(v)$ , radius  $r(G)$  and diameter  $d(G)$  in a connected graph and show that  $r(G) \leq d(G) \leq 2r(G)$  in a connected graph.
3. Show that a graph  $G$  is connected iff  $G$  contains a spanning tree.
4. Define rooted and binary tree. Show that every binary tree  $T$  is a rooted tree.

5. Draw a 11-vertex binary tree.
  - i) With maximum height.
  - ii) With minimum height.
6. Define incidence matrix and write the incidence matrix  $A(G)$  of graph  $G$ .



7. Draw the graph  $K_5$  and find its adjacency matrix.
8. Find the cycle matrix  $C(G)$  of the graph  $G$  shown below. Also find the rank of the cycle matrix  $C(G)$ .

