

Roll No. _____

[Total No. of Pages : 4]

SIS-N-065 B-19
B.Sc. I Semester (CBCS) Degree Examination
MATHEMATICS
Algebra - I and Calculus - I
Paper- BMDSCIT
(New)

Time : 3 Hours

Maximum Marks : 80

Instructions to Candidates: *Answer All the Sections.*

SECTION-A

L Answer any TEN questions. **(10×2=20)**

- 1) Define symmetric and Skew - symmetric matrices.
- 2) Find the rank of the matrix

$$\begin{bmatrix} 1 & -1 & 3 & 2 \\ 1 & 3 & -3 & 4 \\ 5 & 3 & 3 & 9 \end{bmatrix}$$

- 3) Define orthogonal matrix show that the matrix A is orthogonal where

$$A = \frac{1}{6} \begin{bmatrix} -2 & 4 & 4 \\ 4 & -2 & 4 \\ 4 & 4 & -2 \end{bmatrix}$$

- 4) Find the value of λ for which the following system has a non-trivial solution.

$$2x - y + 2z = 0$$

$$3x + y - z = 0$$

$$\lambda x + 2y + z = 0$$

- 5) Verify Cayley - Hamilton theorem for the matrix

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

- 6) Verify the consistency of the system

$$x - y - z = 3$$

$$-x - 10y + 3z = -5$$

$$2x - y + 2z = 2$$

- 7) Find n^{th} derivative of $y = \sin^2 x$.

8) Find $\lim_{x \rightarrow 5} \left\{ \frac{x^2 - 125}{x - 5} \right\}$

- 9) If $y = e^{-x^2}$ then show that $y_{n+1} + 2xy_n + 2ny_{n-1} = 0$.

- 10) State Rolle's theorem and verify for the function $f(x) = x^4$ in $[-1,1]$.

11) Evaluate $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}$

- 12) Find θ for $f(x) = lx^2 + mx + n$ by Lagrange's mean value theorem.

SECTION - B

- II. Answer any THREE questions.

(3×5=15)

- 1) Find the rank of the matrix A by reducing into its normal form

$$A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$$

- 2) Reduce the matrix A into echelon form and find its rank

$$A = \begin{bmatrix} 1 & 4 & 3 & 2 \\ 3 & 2 & 1 & 4 \\ 4 & 6 & 4 & 6 \\ 7 & 8 & 5 & 10 \end{bmatrix}$$

- 3) Define unitary matrix. Show that the matrix $\begin{bmatrix} i & 2i \\ 2i & -i \end{bmatrix}$ is unitary.

- 4) Define Hermitian matrix. Show that the matrix A is Hermitian

$$A = \begin{bmatrix} 5 & 1-i & 2+i \\ 1+i & 9 & -i \\ 2-i & i & 7 \end{bmatrix}$$

SECTION - C

III. Answer any THREE questions. (3×5=15)

- 1) State Cayley - Hamilton theorem and verify it for the matrix

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

- 2) Verify the system

$$x + 2y - z = 3$$

$$3x - y + 2z = 1$$

$$2x - 2y + 3z = 2$$

for consistency and hence solve if it is consistent.

- 3) By using Cayley - Hamilton theorem find A^2 and A^3 of the matrix

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

- 4) Find for what values of λ and μ , the system

$$x + y + z = 6$$

$$2x + 4y + 6z = 20$$

$$x + 2y + \lambda z = \mu$$

has i. A unique solution
ii. Infinite solution
iii. No solution

SECTION - D

IV. Answer any THREE questions. (3×5=15)

- 1) Prove that a function which is continuous in a closed interval is bounded.

- 2) Find n^{th} derivative of

i. $\sinh 3x, \sin 3x$

ii. $\frac{4x}{(x-1)^2(x+1)}$

3) State Leibnitz theorem if $y = \frac{\log x}{x}$, prove that

$$y_n = \frac{(-1)^n n!}{x^{n+1}} \left[\log x - 1 - \frac{1}{2} - \frac{1}{3} - \dots - \frac{1}{n} \right]$$

4) Prove that the limit of a function at a point is unique.

SECTION - E

V. Answer any THREE questions.

(3×5=15)

1) Evaluate

i. $\lim_{x \rightarrow 0} \left[\frac{1}{x^2} - \frac{1}{x \tan x} \right]$

ii. $\lim_{x \rightarrow 0} \left[\frac{\frac{1}{x} - \cot x}{x} \right]$

2) State and prove Lagrange's mean value theorem.

3) i. Obtain the Taylor's expansion of $\tan x$ about $\frac{\pi}{4}$ up to the term containing $\left(x - \frac{\pi}{4}\right)^3$.

ii. Expand $\cos x$ in Maclaurin's series.

4) State Rolle's theorem verify Rolle's theorem for the function $f(x) = e^x \sin x$ over $[0, \pi]$.