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SIVS 208 A-2K12

B.A./B.Sc. IVth Semester Degree Examination
Mathematics
Algebra - III
Paper - 4.1

Time : 3 Hours Maximum Marks : 60

Instructions to Candidates :

*Answer **all** the Sections.*

Section - A

I. Answer **any ten** of the following : **(10×2=20)**

1) Define subgroup of a group and give an example.
2) Find the orders of the elements of the multiplicative group $G = \{1, -1, i, -i\}$
3) Intersection of any two normal sub - groups of a group is also a normal sub-group.
4) Let $f : G \to G'$ be a homomorphism from the group $(G, \cdot)$ into the group $(G', *)$ then show that $f(e) = e'$ where e and e' are identity elements of the group G and G' respectively.
5) Define ring with zero divisors and without zero divisors
6) In a ring $(R, +, \cdot)$ prove that $a(-b) = (-a)b = -(ab) \forall a, b \in R$
7) Show that the set $S = \left\{ \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} : a, b \in z \right\}$ is a sub - ring of the ring $M_2(z)$ for all 2×2 matrices over the set of integers.
8) In a vector space V over the field F, Show that

   $C_1\alpha = C_2\alpha$ and $\alpha \neq 0 \Rightarrow C_1 = C_2$
9) Express the vector (2, -5, -1) as a linear combination of the vectors (1,2,3), (2,1,1), (1,3,2).
10) Define linear independence and linear dependence.
11) Determine whether the set {(1,1,2), (1,2,5), (5,3,4) is a basis of $V_3(R)$.
12) If $T : V_1(R) \to V_3(R)$ defined by $T(x) = (x, x^2, x^3)$ verify whether T is linear or not.

Section - B

II. Answer **any three** of the following : **(3×5=15)**

1) If H and K are any two subsets of group G. then show that $(HK)^{-1} = K^{-1}H^{-1}$
2) Prove that a sub-group H of a group G is normal if and only if $gHg^{-1} = H \ \forall \ g \in G$
3) State and prove legrange's theorem, i.e. O(H) divides O(G) where H is the sub - group of a group G.
4) If $f : G \to G$ be a homomorphism from the group G into itself and H is a cyclic sub-group of G then f(H) is again a cyclic sub-group of G.

Section - C

III. Answer **any two** of the following : **(2×5=10)**

1) A ring is without zero divisors if and only if the cancellation laws holds in it
2) A non - empty sub-set S of a ring R is a sub-ring of R if and only if
   i) S+(-S) = S
   ii) SS ≤ S
3) If R is a ring such that $a^2 = a \ \forall \ a \in R$ then show that
   i) The additive inverse of each element of R is it self.
   ii) $a + b = 0 \Rightarrow a = b$
   iii) R is a commutative ring

Section - D

IV. Answer **any three** of the following : **(3×5=15)**

1) Prove that a non-empty sub-set W of a vector space V is a sub-space of V if and only if
   i) $\alpha, \beta \in W \Rightarrow \alpha + \beta \in W$
   ii) $C \in F, \alpha \in W \Rightarrow C.\alpha \in W$
2) The set $\{(x_1, x_2, x_3)\ (y_1, y_2, y_3)(z_1, z_2, z_3)\}$ of vectors of the vector space V_3(R) is linearly dependent if and only if

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} = 0$$

3) Show that the set $B = \{(1,1,0)(1,0,1)(0,1,1)$ is a basis of the vector space V_3(R)
4) Find the range, null space, rank and nullity of the linear transformation $T : V_3(R) \to V_2(R)$ defined by $T(x, y, z) = (y - x, y - z)$ and also verify Rank - nullity theorem.
