

SIIS 194 B-14
B.A/B.Sc IIIrd Semester Degree Examination
Mathematics
(Real Analysis)
Paper - 3.2

Time : 3 Hours

Maximum Marks : 60

Instructions to candidates:

Answer all questions and compulsorily mention the sections .

SECTION-A

I Answer any ten of the following (10×2=20)

1) Verify Rolls theorem If

$$f(x) = \sqrt{x(x-1)} \text{ in the interval } [0, 1]$$

2) Write the statement of Taylor's theorem with lagranges form of remainder.

3) Verify lagranges mean value theorem if $f(x) = lx^2 + mx + n$ in the interval $[a,b]$

4) Evaluate $\lim_{x \rightarrow b} \frac{x^b - b^x}{x^x - b^b}$

5) Expand $e^{x \cos x}$ upto the term containing x^2 by Maclaurins Expansion

6) Give the definition of norm of partition p and suitable example.

7) If $f(x) = 2x-1$, $0 \leq x \leq 1$ and partition $p = \{ 0, \frac{1}{3}, \frac{2}{3}, 1 \}$ Find $U(p,f)$ and $L(p,f)$

8) If $f,g \in R[a,b]$, then P.T $f.g \in R[a,b]$

9) Evaluate $\int_1^a \int_1^b \frac{dx dy}{xy}$

10) Evaluate $\iiint_0^1 x^2 y z dx dy dz$

- 11) Define Triple integral of $F(x)$
- 12) Define Leibnitz's Rule for differentiation under integral sign

SECTION - B

II Answer any two of the following $(2 \times 5 = 10)$

- 1) State and explain Rolle's Theorem
- 2) Verify the Cauchy's mean value theorem for the functions

$$f(x) = \sqrt{x}, \quad g(x) = \frac{1}{\sqrt{x}} \text{ in } [a, b]$$

- 3) Expand the function $\log_e(1-x)$ up to the term containing x^4 by Maclaurins expansion.

SECTION - C

III Answer any three of the following $(3 \times 5 = 15)$

- 1) If $f(x) = x^2$ is defined on $[0, 1]$ and $p = \left\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\right\}$ find $L(p, f)$ and $U(p, f)$
- 2) If $f, g \in R[a, b]$ and there exists $\lambda > 0$ such that $|g(x)| \geq \lambda$ for all $x \in [a, b]$ Then prove that $\left(\frac{f}{g}\right) \in R[a, b]$
- 3) If $f(x)$ is a monotonic function on $[a, b]$ Then prove that f is R - integrable on $[a, b]$
- 4) Using the substitution $x = \pi - t$ show that

$$\int_0^\pi x \phi(\sin x) dx = \frac{\pi}{2} \int_0^\pi \phi(\sin x) dx$$

SECTION - D

IV Answer any three of the following $(3 \times 5 = 15)$

- 1) Evaluate $\int_C [(3x - 2y)dx + (y + 2x)dy - x^2 dz]$

Where C is the curve $x = t, y = 2t^2, z = 3t^3$ and $0 \leq t \leq 1$

- 2) Evaluate $\int_0^a \int_0^{\sqrt{x^2 - x^2}} \sqrt{d^2 - x^2 - y^2} dy dx$ by changing the order of integration.

3) Evaluate $I = \int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dx dy dz}{\sqrt{a^2-x^2-y^2-z^2}}$

4) Evaluate $\int_0^\infty \frac{\tan^{-1} x}{x(1+x^2)} dx$ ($a > 0$) by applying differentiation under integral sign and

hence $\int_0^\infty \frac{\tan^{-1} ax}{x(1+x^2)} dx$
