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**SIIS - N 195 B - 16**  
**B.A./B.Sc. IIIrd Semester Degree Examination**  
**Mathematics**  
**(Groups, Rings, Line and Multiple Integrals)**  
**Paper : 3.1**  
**(New)**

Time : 3 Hours

Maximum Marks : 60

**Instructions to Candidates :**

Answer **all** questions from all Sections.

**Section - A**

**I** Answer any “TEN” of the following **(10×2=20)**

- 1) Write the associative law and commutative law of a group  $G$  under binary operation “.”
- 2) Show that the multiplicative group of fourth root of unity  $\{1, -1, i, -i\}$  is a cyclic group.
- 3) Define the order of an element of a group  $G$ .
- 4) State “FERMATS” Theorem
- 5) Define the Isomorphism of two groups  $G$  &  $G'$ .
- 6) Define the Kernel of homomorphism
- 7) Write the left & write distributive laws of ring  $R$ .
- 8) Define the field
- 9) Evaluate  $\int_c [x-y] dx + [x+y] dy$  along the Curve  $c: x=t^2; y=t$  where ;  $0 \leq t \leq 1$ .
- 10) Evaluate  $\int_c (2y+x) dx + (3x-y) dy$  along the Curve  $x=2t; y=t^2$  where ;  $0 \leq t \leq 1$ .
- 11) Evaluate  $\int_0^a \int_0^b [x^2 + y^2] dx dy$
- 12) Evaluate  $\int_0^1 \int_1^2 [x^3 + y^3] dy dx$

**Section - B**

**II** Answer any Five of the following **(5×5=25)**

- 1) State and Prove the necessary and sufficient Condition for a sub Group “H” of a group ‘G’
- 2) If  $a, b \in G$  then Prove the following results.
  - i) If  $a \in G$ , then prove that  $(a^{-1})^{-1} = a$
  - ii) If  $a, b \in G$ , then Prove that  $(ab)^{-1} = b^{-1}a^{-1}$
- 3) State and Prove “LAGRANGE’S” Theorem.

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- 4) Prove that the set  $G/H$  of all cosets of a Normal. Sub group  $H$  of the Group  $G$  is a group under the Binary operation defined by.  $Ha.Hb = Hab; \forall Ha, Hb \in G/H$
- 5) Let  $f: G \rightarrow G'$  be a Homomorphism of a groups with Kernel 'K'. Then Prove the following.
  - i) K is sub-group of G.
  - ii) K is normal Sub-grouping
- 6) Let  $(R^{+}, +)$  = Group of all real numbers With Respect to addition  $(R^{+}, \cdot)$  = Group of all positive real numbers with respect to Multiplication then Show that the Mapping  $f: R_{+} \rightarrow R_{+}$  defined by  $f(x) = e^x$ 
  - i) f is homomorphism.
  - ii) f is 1-1
  - iii) f is onto.
- 7) Show that the set  $Z_6 = \{0,1,2,3,4,5\}$  has
  - i) Additive Inverse find?
  - ii) Satisfies commutative law with respect to addition & multiplication modulo 6,x
- 8) Prove that Intersection of two subrings is again a subring of R

**Section - C**

**III.** Answer any **THREE** of the following

**(3×5=15)**

- 1) Evaluate  $\int_C [x^2 - 2y] dx + [x^2 y + 3] dy$ . Around the boundary of the region defined by  $y^2 = 4x$  and  $x = 2$ .
- 2) Prove that  $\int_C xydx + yzdy + zxdz$  Where  $C$  is the curve defined by  $x = t; y = t^2; Z = t^3$  and  $-1 \leq t \leq 1$  is  $-10/7$ .
- 3) Prove that 
$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dx dy dz}{\sqrt{a^2-x^2-y^2-z^2}} = \frac{\pi a^2}{8}$$
- 4) Find the area of a circle by double integration method whose equation is given by  $x^2 + y^2 = 9$
- 5) By applying the Leibnitz's rule for the differentiation under the integral sign prove that  $\int_0^{\infty} \frac{x-1}{\log x} dx = \log(1+\infty)$  where " $\infty$ " is any parameter.