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SIIS 73 A-2K14

B.A./B.Sc. IInd Semester Degree Examination

Mathematics

(Calculus -II)

Paper - 2.2

Time :3 Hours Maximum Marks : 60

Instructions to Candidates:

Answer all sections

Section - A

I. Answer any **ten** of the following: **(10×2=20)**

1) Evaluate $\int \frac{dx}{x^2+2x+6}$

2) Evaluate $\int \frac{x}{(1+x)(2+x)} dx$

3) Evaluate $\int Sin^{-1}x \, dx$

4) Evaluate $\int_0^4 \frac{dx}{\sqrt{16-x^2}}$

5) Show that $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ If f(x) is even function.

6) Evaluate $\lim_{n\to\infty} \sum_{r=1}^{n-1} \frac{n^2}{(n+r)^3}$

7) Evaluate $\int_0^{\pi/2} \sin^4 x \cos^2 x \, dx$

8) Find the Area included between the parabola $y^2 = 4ax$ and its latus rectum.

9) If $u = \log(x^2+y^2)$ then show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

10) If $u = Sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

11) If $Z = x^2+y^2$ where $x = e^t \cos t$, $y = e^t \sin t$ then find $\frac{dz}{dt}$

12) Define Jacobian.

Section - B

II. Answer any five of the following **(5×5=25)**

1) Evaluate $\int \frac{dx}{2+\cos x-\sin x}$

2) Show that $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} = \frac{\pi^2}{4}$

3) Evaluate $\int_0^1 (2x+1)dx$ as a limit of a sum.

4) Obtain the reduction formulae for $\int \sin^n x dx$ where n is positive integer. Hence Evaluate $\int_0^{\pi/2} \sin^6 x dx$

5) Find the length of the arc of the parabola $y^2 = 4ax$ cut off by the latus rectum.

6) Find the area of the Segment cut off from the parabola $y^2 = 2x$ by the straight line $y = 4x-1$

7) Show that the Surface area of the Solid generated by the revolution of the Lemniscate $r^2 = a^2 \cos 2\theta$ about the initial line is $2\sqrt{2}\pi a^2\left(\sqrt{2}-1\right)$

Section - C

III. Answer any three of the following. **(3×5=15)**

1) If $u = \log\sqrt{x^2+y^2+z^2}$ then show that $\left(x^2+y^2+z^2\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)=1$

2) If $Z = f(x,y)\, x = e^u + e^v$ and $y = e^{-u} - e^v$ then show that $\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}$

3) If $u = x+\frac{y^2}{x}$ and $v = \frac{y^2}{x}$ then find $\frac{\partial(u,v)}{\partial(x,y)}$

4) If $x = r\cos\theta$ $y = r\sin\theta$ and $z = z$ then find $\frac{\partial(x,y,z)}{\partial(r,\theta,z)}$
