

Roll No. _____

[Total No. of Pages : 3

SIS 068 B-2K13

B.A./B.Sc. Ist Semester Degree Examination

Mathematics

(Calculus-I)

Paper - 1.2

Time : 3 Hours

Maximum Marks : 60

Instructions to Candidates:

Answer all questions.

Section - A

L Answer any ten of the following:

(10×2=20)

1) If $f(x) = \begin{cases} x^2 + 3 & \text{if } x \leq 1 \\ x + 1 & \text{if } x \geq 1 \end{cases}$ find the limit of $f(x)$ as x tends to 1, if it exists.

2) Examine the differentiability of the function $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$ at $x=0$.

3) Find the n^{th} derivative of $y = \log(ax+b)$ w.r.t x .

4) Find $\tan \phi$ if $\frac{2a}{r} = 1 - \cos \theta$.

5) Show that the curves $r = ae^{\theta}$, $b = re^{\theta}$ intersect orthogonally.

6) Find the pedal equation of the curve $r^2 = a^2 \cos 2\theta$.

7) Find ds/dt if $x = a \cos t$, $y = b \sin t$.

- 8) Find the envelope of the family of curves $x \cos \alpha + y \sin \alpha = p$, α being the parameter.
- 9) Show that $y=e^x$ is concave upwards everywhere.
- 10) Find the asymptotes parallel to the coordinate axes for the curve $x^2 y^2 - y^2 = 2$.
- 11) Define the terms Node, Cusp of a double point.
- 12) If $y' = \sin 4x \sin 2x$. Find y_2 .

Section - B

II. Answer any **four** of the following: **(4×5=20)**

- 1) State and prove Leibnitzs rule to find the n^{th} derivatives of the product of two functions.
- 2) Find n^{th} derivatives of $e^x \cos^2 x \sin x$ w.r.t x .
- 3) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ show that $(1-x^2)y_{n+2} - (2n+3)xy_{n+1} - (n+1)^2 y_n = 0$.
- 4) Find the slope of the tangent at any point (r, θ) on the curve $r = a(1 + \sin \theta)$. Further show that the tangent at the point $\theta = \frac{\pi}{2}$ is parallel to the initial line.
- 5) Show that the pairs of curves $r = a(1 + \sin \theta)$, $r = b(1 - \sin \theta)$ intersect orthogonally.
- 6) Find the p-r equation of the curve $x = a \cos^3 \theta$, $y = a \sin^3 \theta$.

Section - C

III. Answer any **four** of the following: **(4×5=20)**

- 1) Show that the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the radius of curvature is given by $\rho = \frac{a^2 b^2}{p^3}$, where p is the length of the perpendicular from the centre upon the tangent at (x, y) .

- 2) Find the coordinates of the centre of curvature at (x, y) for the curves
 $x = a(t + \sin t), y = a(1 + \cos t)$
- 3) Find the envelope of the family of ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where a and b are parameters connected by the relation $ab = c^2$ where ' c ' is a known constant.
- 4) Find all the asymptotes of the curve $x^3 + y^3 - 3axy = 0$.
- 5) Find the position and nature of the double points on the curve $x^3 + y^3 = 3axy$.
- 6) Trace the curve catenary $y = c \cosh\left(\frac{x}{c}\right), c > 0$.

