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PGIS - N 1024 B - 15

M.Sc. Ist Semester (CBCS) Degree Examination

Mathematics

(Discrete Mathematics)

Paper - HCT 1.4

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates

1. Answer any **five** questions
2. All questions carry **equal** marks.

1. a) Define a poset and a lattice, For any a,b,c,d in a lattice (h,≤), if $a \le b$ *and* $c \le d$, then show that $a \vee c \le b \vee d$ *and* $a \wedge c \le b \wedge d$. **(6)**

b) Prove that in a distributive lattice, if an element has a complement then this complement is unique. **(5)**

c) If the meet operation is distributive over the join operation in a lattice, then prove that, the join operation is also distributive over the meet operation. **(5)**

2. a) Design the digital network for the Boolean expression $\left[(x_1 \vee x_2) \wedge (x_1' \vee x_3)\right] \vee (x_2 \vee x_3)'$ and also write the logic table for the corresponding combinatorial Circuit **(8)**

b) Show that the function f given by the following table is a Boolean function

x_1	x_2	x_3	f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

(4)

c) Design a switching circuit to the statement $(a \wedge b) \vee (a \wedge c) \vee (b \wedge c)$ **(4)**

3. a) Determine how many distinguishable permutations of the letters in the word BANANA. **(5)**

b) If $A_1, A_2, \ldots, An$ are pairwise disjoint sets, then prove that $\left|\bigcup_{i=1}^{n} A_i\right| = \sum_{i=1}^{n} |Ai|$ **(6)**

c) A Computer Company wants to hire 25 programmers to handle Systems Programming jobs and 40 Programmers for applications programming. of those hired, ten will be expected to perform jobs of both types. How many programmers must be hired. **(5)**

4. a) Solve the recurrence relation $a_r - 7a_{r-1} + 10a_{r-2} = 0$ **(5)**

b) Find the particular solution of $a_r + 5a_{r-1} + 6a_{r-2} = 42.4^r$ **(5)**

c) Solve the recurrence relation $a_r = 3a_{r-1} + 2,\ r \geq 1$ With the boundary conditions a_0=1, by the method of generating functions. **(6)**

5. a) Define **(5)**

i) Directed graph

ii) Parallel edges

iii) Weighted graph. Give example for each.

b) Prove that the number of vertices of odd degree in a graph is always even. (5)

c) Prove that in a simple graph with n vertices and k components can have atmost $(n-k)(n-k+1)/2$ edges. (6)

6. a) If A is the adjacency matrix of a simple graph, then show that, the ij^{th} entry of A^n is the number of edge sequences of length n from vertex i to vertex j (6)

b) Prove that a connected graph G is Eulerian if and only if all vertices of G are of even degree. (6)

c) Define transport network with an example. (4)

7. a) Define

i) Semigroup

ii) Monoid. If f is an isomorphism from a semigroup $(S,*)$ to Semigroup $(T,*')$, the prove that f^{-1} is an isomorphism from $(T,*')$ *to* $(S,*)$ (6)

b) Prove that a subgroup H of a group G is normal in G if and only $aHa^{-1} = H$ for every a in G. (5)

c) State and Prove Lagranges theorem on groups of finite order (5)

8. a) Define the weight of a code word. Find the weight of each of the following words in B^6

i) $x = 100101$

ii) $x = 011001$

iii) $x = 101010$

iv) $x = 111000$ (4)

b) Find the minimum distance of the (3,8) encoding function $e: B^3 \to B^8$ defined by

e (000) = 00000000

e (001) = 01110010

e (010) = 10011100

e (011) = 01110001

e (100) = 01100101

e (101) = 10110000

e (110) = 11110000

e (111) = 00001111

How many errors will e detect. **(6)**

c) Show that (3,6) encoding function $e: B^3 \to B^6$ defined by

e (000) = 000000

e (001) = 001100

e (010) = 010011

e (011) = 011111

e (100) = 100101

e (101) = 101001

e (110) = 110110

e (111) = 111010

is a group code . Find the minimum distance of the group code. **(6)**
