PGIVS 1311 A-2K12

M.A./M.Sc. Fourth Semester Degree Examination

MATHEMATICS — Paper – 4.4

Graph Theory – II (Elective B)

(Old)

Time : 3 Hours] [Max. Marks : 80

Instructions : *1) Answer any **five** full questions.*

*2) All questions carry **equal** marks.*

1. (a) If G is a maximal planar (p, q) graph with $p \geq 3$, then prove that $q = 3p - 6$. **(6)**

   (b) Show that for any graph G, $K(G) \leq \lambda(G) \leq \delta(G)$. **(10)**

2. (a) Prove that a graph G is outerplanar if and only if it has no subgraph homeomorphic to K_4 or $K_{2,3}$. **(10)**

   (b) Show that a graph G is planar if and only if it has no subgraph homeomorphic to K_5 or $K_{3,3}$. **(6)**

3. (a) Give relationship between a planar graph G and its dual. Further show that for any connected plane graph G is isomorphic to its double dual. **(10)**

   (b) Write the steps for detection of planarity. **(6)**

4. (a) Prove that every tree with two or more vertices is a 2-chromatic. **(8)**

   (b) Show that a graph G is bicolorable if and only if it has no odd cycles. **(8)**

5. (a) Prove that for any graph G, $x(G) \leq 1 + \max \delta(G')$ where maximum is taken over all induced subgraphs G' of G. **(6)**

   (b) Show that for any graph G, $x(G) \leq \Delta(G) + 1$. **(10)**

6. (a) Define uniquely colorable graph. Prove that in n-coloring of a uniquely n-colorable graph, the subgraph induced by the union of any two color classes is connected. **(8)**

   (b) Show that for any K-regular bipartite graph with $K > 0$, G has a perfect matching. **(8)**

7. (a) Show that every acyclic digraph G has atleast one vertex with zero in-degree and atleast one zero out-degree. **(8)**

(b) Prove that a tournament is transitive if and only if it is acyclic. **(8)**

8. (a) Show that for any connected graph G, $\left\lceil \frac{diam(G)+1}{3} \right\rceil \leq \gamma(G)$. **(6)**

(b) Prove that for any tree T, $\gamma(T) = n - \Delta(T)$ if and only if T is a wounded spider. **(10)**
