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# PGIS-N 1022 B-2K13

## M.A./M.Sc. Ist Semester (CBCS) Degree Examination

## Mathematics

## (Ordinary Differential Equations)

## Paper - HCT - 1.3

## (New)

Time : 3 Hours

Maximum Marks : 80

***Instructions to candidates:***

1) *Answer any **five** questions.*
2) *All questions carry **equal** marks.*

1. a) For any real $x_0$, and constants $\alpha, \beta$ then prove that there exists a solution $\phi$ of the initial value problem $y'' + a_1 y' + a_2 y = 0$ on $-\infty < x < \infty$ satisfying $y(x_0) = \alpha, y'(x_0) = \beta$. **(8)**

   b) Let $\phi$ be any solution of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing a point $x_0$, then prove that for all x in I $\|\phi(x_0)\| e^{-k|x-x_0|} \le \|\phi(x)\| \le \|\phi(x_0)\| e^{k|x-x_0|}$ Where $\|\phi(x)\| = \left[ |\phi(x)|^2 + |\phi'(x)|^2 \right]^{1/2}$, $k = 1 + |a_1| + |a_2|$ **(8)**

2. a) Let $\alpha_1, \alpha_2, \ldots\ldots, \alpha_n$ be any n constants. Let $x_0$ be any real number on any interval I containing $x_0$, then prove that there exist at most one solution $\phi$ of the initial value problem L(y)=0 satisfying. $\phi(x_0) = \alpha_1, \phi'(x_0) = \alpha_2, \ldots\ldots, \phi^{(n-1)}(x_0) = \alpha_n$. **(8)**

   b) Let $\phi_1, \phi_2, \ldots\ldots, \phi_n$ be 'n' solutions of L(y)=0 on an interval I, then prove that

$$W(\phi_1, \phi_2, \ldots, \phi_n)(x) = \exp\left[ -\int_{x_0}^{x} d_1(t)\, dt \right] W(\phi_1, \phi_2, \ldots \phi_n)(x_0).$$ **(8)**



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3. a) Explain the method of finding Green's Function of $L_y = P\frac{d^2y}{dx^2} + \frac{dp}{dx}.\frac{dy}{dx} + qy = 0$ where L is the differential operator with boundary conditions $\begin{matrix} \alpha y(a) + \beta y'(a) = 0 \\ \gamma y(b) + \delta y'(b) = 0 \end{matrix}$. **(8)**

b) Find the Green's function corresponding to differential operator $L = \frac{d^2}{dx^2}$ with boundary conditions y(0)=0,y(1)=0. **(8)**

4. a) Define Eigen values and Eigen functions of the sturm Liouville problem $\frac{d^2y}{dx^2} + \lambda y = 0$. **(4)**

b) Find the Eigen values and Eigen functions of the sturm Liouville's problem $\frac{d}{dx}\left[\left(x^2+1\right)\frac{dy}{dx}\right] + \left(\frac{\lambda}{x^2+1}\right)y = 0$ with y(0)=0=y(1). **(12)**

5. a) Transform the following equations into an equivalent self-adjoint equation.

i. $x^2.y'' - 2xy' + 2y = 0$

ii. $f(x).y'' + g(x)y' = 0$ **(8)**

b) If u(x) and *v(x)* be real linearly independent solutions of $[r(x)y']' + P(x)y = 0$ on $a \le x \le b$ then prove that between two consecutive zeros of u(x) there is exactly one zero of *v(x)* and vice-versa. **(8)**

6. a) Let *y(x)* and *z(x)* be non-trivial solution of $[r(x)y']' + P(x)y = 0$.......(1) and $[r(x)z']' + q(x)z = 0$......(2) where $r(x)>0$; $r(x), p(x)$ and $q(x)$ are continuous on $a \le x \le b$ and $q(x)>p(x)$. If the solution y(x) of the equation (1) has consecutive zeros at $x=x_0$, and $x=x_1 (x_0<x_1)$, prove that a solution $z(x)$ of equation (2) which vanishes at $x=x_0$ will vanish again on the interval $x_0<x<x_1$. **(10)**

b) Obtain the formal expansion of the function f, where $f(x) = \pi x - x^2, 0 \le x \le \pi$, in the series of orthonormal characteristic function $\{\phi_n\}$ of the sturm-Liouville problem $\frac{d^2y}{dx^2} + \lambda y = 0$, with $y(0) = 0, y(\pi) = 0$. **(6)**



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7. a) Derive picard's iterative method for obtaining the approximate solution of the problem, $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$. **(8)**

b) Find the third approximation of the equation $\frac{dy}{dx} = z, \frac{dz}{dx} = x^3(y+z)$ by picard's method where $y = 1, z = \frac{1}{2}$ when $x=0$. **(8)**

8. a) Show that there are two values of k for which $k/x$ is an integral of $x^2(y_1 + y^2) = 2$, and hence obtain the general solution. **(5)**

b) Explain the method of solving the Riccati's equation $y_1 = P + Qy + Ry^2$. Where P,Q,R are functions of x, when its one particular integral is known. **(5)**

c) Find the general solution of the equation $x^2 y_1 + 2 - 2xy + x^2 y^2 = 0$. **(6)**



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