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PGIS-N 1029 B-2K13

M.A./M.Sc. Ist Semester (CBCS) Degree Examination

Mathematics

(General Topology)

Paper - HCT-1.5

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to candidates:

1. *Answer any **five** questions.*
2. *All questions carry **equal** marks.*

1. a) Define the usual topology U on the set of reals $\mathbb{R}$. By using the axioms of a topology, show that U is a topology on $\mathbb{R}$. **(8)**

b) If A and B are the subsets of a topological space X then prove the following.

i) $\overline{A}$ is the smallest closed set containing A.

ii) A is closed iff $A=\overline{A}$.

iii) If $A \subset B$ then $\overline{A} \subset \overline{B}$.

iv) $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

v) $\overline{(\overline{A})}=\overline{A}$. **(8)**

2. a) If A is a subset of a space X. Then prove the followings.

i) $A \cup D(A)$ is closed.

ii) $\overline{A} = A \cup D(A)$ where D(A) is the derived set. (8)

b) Define base for a topology. Then prove the following two properties on a base β are equivalent.

i) β is a base for τ.

ii) For each $G \in \tau$ and each $P \in G$ there is $U \in \beta$ such that $P \in U \subset G$. (8)

3. a) Let f be a mapping of a space X into space Y and S be the subbase for the topology on Y then prove the followings are equivalent.

   i. f is continuous.

   ii. The inverse image of each member of S is open in X. **(8)**

   b) If X,Y are the topological spaces and $f : X \to Y$ be a mapping then prove that f is closed iff $\overline{f(A)} \subset f(\overline{A})$ for $A \subset X$. **(8)**

4. a) Define a T_2-space. If X is T_2-space and $f : X \to Y$ is a closed bijection. Then show that Y is T_2-space. **(8)**

   b) Show that every completely normal space is normal and hence every T_5-space is a T_4-space. **(8)**

5. a) State the two axioms of countability. Prove that every 2^0-countable space is 1^0- countable. Is the converse true? Justify your answer. **(8)**

   b) Define subsequence of a sequence. Let $F : N \to X$ converges to 'a' in X then every subsequence of f in X converges to 'a'. **(8)**

6. a) Define a connected space. Show that any continuous image of a connected space is connected. **(8)**

   b) Prove that in any connected space,

   i) Every component of a space X is a maximal connected set.

   ii) The components are closed. **(8)**

7. a) Define compact topological space. Let A be a compact subset of Hausdorff space X and $P \notin A$ then show that there exists disjoint open sets U & V such that $P \in V$ and $A \subset U$. **(8)**

   b) Let X be a compact Hausdorff space and Y be an arbitrary space. If $f : X \to Y$ is a continuous closed surjection then prove that Y is also Hausdorff. **(8)**

8. a) Prove that in any metric space the set of all open spheres is a base for topology on X. **(8)**

   b) Define a Lindelcf space. Show that every closed subspace of Lindelof space is Lindelof. **(8)**
