PGIS-N 1029 B-2K12

M.A./M.Sc. First Semester (CBCS) Degree Examination

MATHEMATICS — Paper – SCT 1.1

Classical Mechanics

(New)

Time : 3 Hours] [Max. Marks : 80

Instructions : *1) Answer any **five** questions.*

*2) All questions carry **equal** marks.*

1. (a) State and prove D'Alembert's principle.

   (b) By using generalized Lagrange's equations derive Lagrange's equation for impulsive motion.

2. (a) Construct Lagrangian and hence the equation of motion of a simple pendulum placed in a uniform gravitational field.

   (b) Derive Hamilton's canonical equations.

3. (a) Obtain Hamilton's equation of motion of a particle in a plane referred to moving axes, where the components of velocities are $u = \dot{x} - \omega y$, $v = \dot{y} + \omega x$.

   (b) Show that $M z = \partial L / \partial \dot{\varphi}_i$ for a system with

   $$L = \frac{1}{2}\sum_i m_i \left(\dot{r}_i^2 + r_i^2 \varphi_i^2 + \dot{z}_i^2\right) - U(r).$$

4. (a) Deduce Lagrange and Hamilton equations from Hamilton's principle.

   (b) Derive Hamilton's canonical equation from Hamilton's principle.

5. (a) Derive Whittaker's equation.

   (b) Prove that Poisson's bracket of two constants of motion is itself a constant of motion.

6. (a) Show that Poisson brackets are also invariant under canonical transformation.

   (b) Prove that the transformation given by $q = \sqrt{2P}\sin Q$, $p = \sqrt{2P}\cos Q$ is canonical by using Poisson brackets.

7. (a) Find the value of q and p for a harmonic oscillator described by the Hamiltonian $H = \frac{1}{2}\left(p^2 + \omega^2 q^2\right)$ and generated by the function $F = \frac{1}{2}\omega q^2 \cot 2\pi Q$.

(b) Derive Hamilton–Jacobi equation.

8. (a) Show that Lagrange's bracket is invariant under canonical transformation.

(b) Prove the following :

(i) $\{q_i, q_j\} = 0$

(ii) $\{p_i, p_j\} = 0$

(iii) $\{q_i, p_j\} = \delta_{ij}$.