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PGIS-N 1026 B-2K13

M.A./M.Sc. Ist Semester (CBCS) Degree Examination
Mathematics
(Classical Mechanics)
Paper - SCT-1.1
(New)

Time : 3 Hours   Maximum Marks : 80

Instructions to Candidates:

1. Answer any **five** questions
2. All questions carry **equal** marks

1. a) Show that the total work done by the effective forces is zero in an infinitesimal virtual displacement, compatible with the constraints of any dynamical system. **(8)**

   b) Derive Lagrange's equation for impulsive motion **(8)**

2. a) Construct the Lagrangian and equations of motion of a spherical pendulum placed in a uniform gravitational field. **(8)**

   b) Derive Hamilton's canonical equations **(8)**

3. a) Obtain Hamilton's equation of motion of a particle in a plane referred to moving axes, where the components of velocities are $u = \dot{x} - \omega y, v = \dot{y} + \omega x$ **(8)**

   b) Obtain Euler's dynamical equations of the motion of a rigid body about a fixed point on the body. **(8)**

4. a) Explain Eulerian angles with a neat diagram **(8)**

   b) Show that $M_z = \frac{\partial L}{\partial \dot{\phi}_i}$ for a system with L = $\frac{1}{2}\sum_i m_i(\dot{r}_i^2 + r_i^2\dot{\phi}_i^2 + \dot{Z}_i^2) - U(r)$ **(8)**

5. a) A symmetrical top can turn freely about a fixed point in its axis of symmetry and is acted on by forces derived from the potential function $\mu \cot^2 \theta, \theta$ is the angle between this axis and a fixed line, say 0Z. Show that, the equation of motion can be integrated in terms of elementary functions. (8)

b) Deduce Lagrange and Hamilton equations from Hamiltons principle (8)

6. a) Derive poincare integral invariant (8)

b) A particle of mass m is falling under gravity. Solve for motion of the particle using the canonical transformation theory (8)

7. a) Show that poisson brackets are also invariant under canonical transformation .(8)

b) Find the value of q and p for a harmonic oscillator described by the Hamilton $H = \frac{1}{2}(P^2 + \omega^2 q^2)$ and generated by the function $F = \frac{1}{2}\omega q^2 \cot 2\pi Q$ (8)

8. a) Deduce Hamilton -Jacobi equation (8)

b) Show that Lagrange's bracket is invariant under canonical transformation (8)
