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PGIS-N 1020 B-15

M.A/M.Sc Ist Semester (CBCS) Degree Examination

Mathematics

(Algebra - I)

Paper - HCT 1.2

(New)

Time : 3 Hours Maximum Marks : 80

Instructions to Candidates

1. Answer any **five** full questions
2. All questions carry **equal** marks.

1. a) Prove that any subgroup of an infinite cyclic group is also infinite cyclic. Also prove that an infinite cyclic. Group has exactly two generators **(8)**

b) Show that every permutation $\sigma \in S_n$, where S_n is the symmetric group and it can be expressed as a product of disjoint cycles. **(8)**

2. a) State and prove Cayley's theorem **(8)**

b) Derive the class equation for finite group. **(8)**

3. a) Prove that a group G is solvable if and only if $\overset{(n)}{G} = \{e\}$ for some $n \geq 1$ **(8)**

b) Show that a group of prime order is solvable. Also prove for a group G with K normal subgroup such that both K and G/K are solvable, then G is solvable. **(8)**

4. Prove that every integrable domain can be embedded in a field. **(16)**

5. a) Let R be an Euclidean domain then show that for any $a \in R$ Which is not a unit can be expressed as a product of irreducible elements **(8)**

b) If R is commutative ring with unit element, then show that R $[x]$ is also commutative ring. If R is an integral domain then prove that R$[x]$ is also an integral domain. **(8)**

6. a) Show that if F is a field, then F $[x]$ is a Euclidean domain (8)

b) Let R be a unique factorisation domain. Then prove that the polynomial ring R $[x]$ is a unique factorisation (8)

7. a) Prove that $F(\alpha)$ has dimension n as a vector space over F. (8)

b) Let K/F and L/K be algebraic extension then prove that L/F is an algebraic extension (8)

8. a) Let $f(x) \in F[x]$ be of degree n. Then show that $f(x)$ has a splitting field. (8)

b) For a finite field F with P^n elements. Prove that F has a subfield F' with P^m elements if and only if m divides n. (8)