

SIVS-N 194 A-16
B.A./B.Sc. IVth Semester Degree Examination
MATHEMATICS
(Vector Analysis & Fourier Transformations)
Paper : 4.2
(New)

Time : 3 Hours

Maximum Marks : 60

Instructions to Candidates:

Answer all the sections.

Section - A

Answer any Ten of the following : (10×2=20)

1. Find the directional derivative of the function $f(x, y, z) = xyz - xy^2z^3$ at $(1, 2, -1)$ in the direction $i + j + 3k$.
2. Define the gradient of a scalar point function and find its magnitude.
3. Prove that the vector field
$$\bar{f} = (x+3y)i + (y-3z)j + (x-2z)k$$
 is SOLENOIDAL.
4. If $\bar{f} = 3xyi + 20yz^2j - 15zxk$ & $\phi = y^2 - zx$, find $\operatorname{div}(\phi \bar{f})$
5. With respect to the position vector \bar{r} give the expression for cylindrical and spherical polar coordinates.
6. The Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$ is $x^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos nx$, For What value of x.

Prove that following result

i) $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \dots$

ii) $\frac{\pi^2}{12} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16}$

7. If $f(x) = x$ in $[0, \pi]$ find the Fourier coefficient b_n .
8. The Fourier series of the function $f(x)$ in the interval $(-\pi, \pi)$ is given by

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos nx + \sum_{n=1}^{\infty} B_n \sin nx$$
 What are the values of A_0 , A_n & B_n .
9. If $f(x) = (x - 1)$ in $(-\pi, \pi)$ then prove that $a_0 = -2$
10. What is a Fourier integral.
11. Give the formula for Fourier Integral of "sine" function.
12. If $f(x) = e^{-x}$ $x > 0$ and $f(-x) = f(x)$ then prove that the fourier integral function

$$e^{-x} = \frac{2}{\pi} \int_0^{\infty} \frac{\cos \alpha x}{\alpha^2 + 1} d\alpha$$

Section - B

Answer any four of the following :

(4×5=20)

1. If ϕ & Ψ be continuously differentiable scalar point function
 - i) Then prove that

$$\text{grad} \left\{ \frac{\phi}{\Psi} \right\} = \frac{\Psi \nabla \phi - \phi \nabla \Psi}{\Psi^2}$$
 - ii) Prove that $\nabla \times (\bar{r} \times \bar{a}) = -2\bar{a}$
2. Show that $\text{div} \left\{ \frac{f(r)}{r} \bar{r} \right\} = \frac{1}{r^2} \frac{d}{dr} \{ r^2 f(r) \}$ where $\bar{r} = xi + yj + zk$
3. Prove that
 - i) $\nabla \cdot (\nabla \times \bar{f}) = 0$
 - ii) $\nabla \times (\nabla \phi) = \bar{0}$
4. If $\bar{f} = (\sin y + z)i + (x \cos y - z)j + (x - y)k$. Prove that \bar{f} is Irrotational. Find the function ϕ such that $\bar{f} = \nabla \phi$.

5. If \vec{r} is the position Vector & $r = |\vec{r}|$ then prove the following result

i) $\operatorname{div} (r^3 \vec{r}) = 6r^3$

ii) If $\vec{f} = xi + yj + zk$

Then prove that $\operatorname{div} \vec{f} = 3$

6. Evaluate by Greens Theorem for $\oint_C [(xy + y^2) dx + x^2 dy]$ Where 'C' is the closed curve of the region bounded by $y = x$ & $y = x^2$
7. By Using Gauss Divergence Theorem show that

$$\iint_S \vec{r} \cdot \hat{n} ds = 3V, \text{ Where 'S' is a closed surface.}$$

Section - C

Answer any four of the following :

(4×5=20)

1. Find the Fourier series for the function $f(x) = \frac{x^2}{4}; -\pi < x < \pi$
2. Find the Fourier series for the function $f(x)$ defined by

$$f(x) = \begin{cases} 0 & -2 < x < 0 \\ 1 & 0 < x < 2 \end{cases}$$

Given $f(x+4) = f(x)$

Only find the fourier coefficients a_0 & a_n .

3. Obtain the Half - range Cosine series for $f(x) = x \sin x$ in $(-\pi, \pi)$
4. If 'a' is any real constant then prove the following results.

i) $F \{f(x+a)\} = e^{-iax} \hat{f}(a)$

ii) $F \{(e^{-iax}) f(x)\} = \hat{f}(a+a)$

5. Using Fourier Integral method show that

$$a,b > 0 \quad \frac{e^{-ax} - e^{-bx}}{2} = \frac{(b^2 - a^2)}{\pi} \int_0^\infty \frac{u \sin ux du}{(u^2 + a^2)(u^2 + b^2)}$$

6. Using Fourier-Integral Method ,Show that

$$\int_0^\infty \frac{\sin \alpha \pi \sin \alpha x}{1 - \alpha^2} d\alpha = \begin{cases} \frac{\pi}{2} \sin x & -\pi \leq x \leq \pi \\ 0 & \text{otherwise} \end{cases}$$
