

Roll No. \_\_\_\_\_

**SIVS-N 194 A-18**  
**B.A./B.Sc. IVth Semester Degree Examination**  
**MATHEMATICS**  
**(Linear Algebra and Laplace Transformation)**  
**Paper : 4.1**  
**(New)**

Time : 3 Hours

Maximum Marks : 60

**Instructions to Candidates:**

*Answer all sections.*

**SECTION - A**

Answer any **TEN** of the following :

**(10×2=20)**

1. Define vector space.
2. Define the linear combination of vector space over the field F.
3. Define the terms linearly dependent and Independent of a Vector space.
4. Prove that if  $S = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$  forms a basis of  $V_4(R)$  and what is the dimension?
5. Define a vector - subspace.
6. Define the linear - Transformation of two vector spaces defined over the same field.
7. Find the Laplace transformation of  $\{\sin t, \cos t, e^{at}\}$ .
8. Find the Laplace transformation of  $(1 + e^t)^2$ .
9. State convolution theorem.
10. Find the Inverse Laplace transform of  $\left\{ \frac{2s^2 + 5s - 8}{s^3} \right\}$ .

11. Find the Laplace transforms of  $\{\sinh at + t^3 + e^{-3t}\}$ .
12. Define Laplace transformation.

### SECTION - B

Answer any **FOUR** of the following.

(4×5=20)

1. Express  $\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$  as a linear combination of  $\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix} \right\}$ .

2. Prove that any subset of linearly independent set is linearly independent
3. Define the terms Basis & Dimension Determine Whether

$S = \{(1, 0, 0), (1, 1, 0), (1, 1, 1), (0, 1, 0)\}$  is a basis of  $V_3(R)$  OR not?

4. Prove that the following is a  $\alpha T$   $T: V_2(R) \rightarrow V_2(R)$  defined by  $T(x, y) = (x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta)$ .
5. Find the range, null spaces;  $s(T), v(T)$  & verify Rank. Nullity Theorem of LT  $T: R^3 \rightarrow R^3$  defined by  $T(e_1) = e_1 - e_2; T(e_2) = 2e_1 + e_3$   $T(e_3) = e_1 + e_2 + e_3$  Where  $e_1 = (1, 0, 0), e_2 = (0, 1, 0), e_3 = (0, 0, 1)$ .

6. Find the matrix of LT  $T: R^2 \rightarrow R^2$  defined by  $T(x, y) = (3x - 4y, x + 5y)$  Relative to the Basis
  - i)  $\{(1, 0), (0, 1)\}$
  - ii)  $\{(1, 3), (2, 5)\}$

### SECTION - C

Answer any **FOUR** of the following.

(4×5=20)

1. If  $L\{f(t)\} = F(S)$  then prove the following
  - i)  $L\{f'(t)\} = SF(S) - f(0)$
  - ii)  $L\{f''(t)\} = S^2F(S) - Sf(0) - f'(0)$

2. Find the Inverse Laplace Transform of  $\{2S / S^2 + S - 2\}$ .

3. Express  $f(t) = \begin{cases} t & 0 < t \leq 2 \\ t^2 & t > 2 \end{cases}$  as unit step function & find Laplace transform.

4. Find the Inverse Laplace Transform of  $\left\{ \frac{s}{(s-3)(s^2+4)} \right\}$ .

5. Solve  $y'' - 9y = -8e^t$  Given  $y(0) = 0$ ;  $y'(0) = 10$ .

6. Solve  $y'' + y = 0$  Given  $y(0) = 1$ ;  $y'(0) = -1$

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