

SIS -N 067 B-16
B.A./B.Sc. Ist Semester Degree Examination
Mathematics
(Calculus - I)
Paper : 1.2
(New)

Time : 3 Hours

Maximum Marks : 60

Instructions to Candidates:

Answer all the sections.

SECTION-A

L Answer any TEN of the following: (10×2=20)

1) Find the n^{th} derivative of $\log(x^2 - 4x + 4)$

2) Find the n^{th} derivative of $\frac{x+3}{(x-1)(x+2)}$

3) Show that $f(x) = \begin{cases} \frac{\sin 5x}{x}, & \text{when } x \neq 0 \\ 5, & \text{When } x = 0 \end{cases}$

is continuous at $x=0$.

4) Examine the differentiability of $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2-x, & x > 1 \end{cases}$ at $x=1$

5) Verify the Lagranges mean value theorem for $f(x) = \tan^{-1} x$ over $[0,1]$

6) Evaluate $\lim_{x \rightarrow 0} \sin x \log x$

7) Expand the function e^x by Maclaurin's Expansion.

8) If $u = \log(x^2 + y^2)$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

9) If $u = \log(x^2 - y^2)$, $x = \cos t$, $y = \sin t$ find $\frac{du}{dt}$.

10) If $u = \log\left(\frac{x^3 + y^3}{x + y}\right)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2$.

11) If $u = \frac{x+y}{2}$, $v = \frac{x-y}{2}$ show that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{-1}{2}$.

12) If $x = r \cos \theta$, $y = r \sin \theta$ show that $\frac{\partial(x,y)}{\partial(r,\theta)} = 1$.

SECTION-B

II. Answer any TWO of the following: (2×5=10)

1) Find the n^{th} derivative of $e^{ax} \sin(bx + c)$.

2) State and prove the Leibnitz theorem.

3) If $y = e^{\sin^{-1} x}$ show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+1)y_n = 0$

SECTION-C

III. Answer any Three of the following: (3×5=15)

1) A functions which is continuous in closed interval then show that it attains its bounds.

2) State and prove Rolle's theorem

3) Verify Cauchy's mean value theorem for $f(x) = e^x$, $g(x) = e^{-x}$ over $[1,3]$.

4) Obtain the Taylor's Expansion of $\tan x$ about $\frac{\pi}{4}$ upto the term containing $\left(x - \frac{\pi}{4}\right)^3$.

5) Evaluate $\lim_{\theta \rightarrow 0} \left(\frac{\sin \theta}{\theta} \right)^{1/\theta}$.

SECTION-D

IV. Answer any Three of the following:

(3×5=15)

1) $u = (x^2 + y^2 + z^2)^{\frac{1}{2}}$ then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 1$.

2) If $u = f(r)$, $x = r \cos \theta$, $y = r \sin \theta$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

3) Verify the Euler's theorem for the function $u = \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}$.

4) If $u = x^2 + y^2 + z^2$, where $x = e^t$, $y = e^t \cosh t$, $z = e^t \sinh t$ find $\frac{du}{dt}$.

5) If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at $(1, -1, 0)$.
