

SIS-N 067 B-15
B.A./B.Sc. Ist Semester Degree Examination
Mathematics
(Calculus - I)
Paper :1.2
(New)

Time : 3 Hours

Maximum Marks : 60

Instructions to Candidates:

Answer All the sections

Section - A

Solve any Ten of the following

(10×2=20)

- Find the n^{th} derivative of $\frac{2x-1}{(x-2)(x+1)}$
- Find the n^{th} derivative of $\sin^4 x$

- If $f(x) = \begin{cases} x^2 - 1 & \text{When } x < 1 \\ 0 & \text{When } x = 1 \\ 1 - 1/x & \text{When } x > 1 \end{cases}$

Show that the function is continuous at $x = 1$

- Examine the differentiability of the function $f(x) = \begin{cases} x^2 & \text{if } x \leq 3 \\ 6x - 9 & \text{if } x > 3 \end{cases}$ at $x = 3$

- Verify the Rolle's theorem for the function $f(x) = x^2 - 6x + 8$

- Evaluate $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}$.

- Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan x}{\tan 3x}$.

8. If $u = x \tan y + y \tan x$ Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

9. If $z = e^x \sin y$ Where $x = \log t, y = t^2$ then find $\frac{dz}{dt}$

10. If $u = \sin^{-1} \left(\frac{x^2 + y^2}{x + y} \right)$ Show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$

11. If $u = 3x + 5y$ and $v = ux - 3y$ Show that $\frac{\partial(u, v)}{\partial(x, y)} = -29$

12. If $u = x + \frac{y^2}{x}$ and $v = \frac{y^2}{x}$ then find $\frac{\partial(u, v)}{\partial(x, y)}$

Section -B

Answer any Two of the following

(2×5=10)

1. Find the n^{th} derivative of $\log(ax + b)$

2. Find the n^{th} derivative of $e^x \sin x \cos 2x$

3. If $y = [x + \sqrt{1+x^2}]^m$ then show that

$$(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$$

Section - C

Answer any Three of the following

(3×5=15)

1. A function $f(x)$ which is differentiable at $x = a$ then it is continuous at $x = a$ is converse true if not give an example

2. State and prove Lagranges mean value theorem.

3. If $x > 0$ then show that $\frac{x}{1+x} < \log(1+x) < x$

4. Expand the function $\log_e(1+x)$ upto term containing x^4 by maclaurens expansion

5. Evaluate $\lim_{x \rightarrow 0} \left[\frac{1 + \cos 2x}{2} \right]^{1/x^2}$

Section - D

Answer any Three of the following

(3×5=15)

1. If $z = (1 - 2xy + y^2)^{-1/2}$ Show that $x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = y^2 z^3$.
 2. If $u = f(r)$ where $r^2 = x^2 + y^2$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$
 3. State and prove Euler's theorem for homogeneous function.
 4. If $f = f(u, v, w)$ and $u = x/y, v = y/z$
 $w = z/x$ then show that $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} + z \frac{\partial f}{\partial z} = 0$
 5. If $x = r \cos \theta, y = r \sin \theta$ Find $J = \frac{\partial(x, y)}{\partial(r, \theta)}$, $J' = \frac{\partial(r, \theta)}{\partial(x, y)}$ Also verify $J J' = 1$
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